Homework 4

due October 30, 2001

(1) Show: If the order of a group G is p^2 where p is prime, then G is abelian.
(Hint: see HW 3 Problem 3)
(2) Dummit, Foote Section 3.2 Exercise 9 (page 97)
(3) Dummit, Foote Section 3.2 Exercise 11 (page 97)
(4) Dummit, Foote Section 3.2 Exercise 16 (page 97)
(5) Dummit, Foote Section 3.4 Exercise 7 (page 103)
(6) Dummit, Foote Section 3.4 Exercise 9 (page 103)
(7) Dummit, Foote Section 4.2 Exercise 10 (page 124)

Extra Problem:

(1) Show that the center of S_4 is $\{1\}$. Conclude that S_4 is isomorphic to the group of all inner automorphisms of S_4.
(2) Find the Sylow 3-subgroups of S_4.
(3) Show that every automorphism of S_4 is an inner automorphism and hence $S_4 \cong \text{Aut}S_4$.
(Hint: Every automorphism of S_4 induces a permutation of the set $\{P_1, P_2, P_3, P_4\}$ of Sylow 3-subgroups of S_4. If $f \in \text{Aut}S_4$ has $f(P_i) = P_i$ for all i then $f = 1_{S_4}$.)