Homework 7

due November 20, 2001

1. Dummit, Foote, Section 5.5 Exercise 11 (page 188)
2. Prove part (1) of the Jordan-Hölder Theorem (pg. 105).
3. A series $1 = N_0 \trianglelefteq N_1 \trianglelefteq \cdots \trianglelefteq N_k = G$ is called a subnormal series. A one-step refinement of this series is any subnormal series of the form $1 = N_0 \trianglelefteq N_1 \trianglelefteq \cdots \trianglelefteq N_i \trianglelefteq N \trianglelefteq N_{i+1} \trianglelefteq \cdots \trianglelefteq N_k = G$. A refinement of a subnormal series is any subnormal series obtained by a finite number of one-step refinements.

   Show that every refinement of a solvable series is a solvable series.
4. Show that a subnormal series is a composition series if and only if it has no proper refinements.
5. Any group of order p^2q (p, q distinct primes) is solvable.
6. If N is a nontrivial normal subgroup of a nilpotent group G, then $N \cap Z(G) \neq \{1\}$.
   [Hint: You may use that a nilpotent group is isomorphic to the direct product of its Sylow subgroups; see Theorem 3 on page 193.]