

Homework 8  
due November 26, 2003

**1. Rosen 7.2 #11, pg. 236**

What is the product of the positive divisors of a positive integer  $n$ ?

**2. Rosen 7.2 #12, pg. 236**

Show that the equation  $\sigma(n) = k$  has at most a finite number of solutions when  $k$  is a positive integer.

**3. Rosen 7.3 #11,13, pg. 248**

Let  $n$  be a positive integer. We say that  $n$  is deficient if  $\sigma(n) < 2n$  and we say that  $n$  is abundant if  $\sigma(n) > 2n$ .

(a) Show that there are infinitely many deficient numbers.

(b) Show that there are infinitely many odd abundant numbers. (*Hint*: Look at integers of the form  $n = 3^k \cdot 5 \cdot 7$ ).

**4. Rosen 7.4 #10, pg. 256**

Show that if  $n$  is a positive integer, then  $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$ .

**5. Rosen 7.4 #22, pg. 257**

Let  $n$  be a positive integer. Show that

$$\prod_{d|n} \mu(d) = \begin{cases} -1 & \text{if } n \text{ is a prime;} \\ 0 & \text{if } n \text{ has a square factor;} \\ 1 & \text{if } n \text{ is square-free and composite.} \end{cases}$$

**6. Rosen 8.4 #8, pg. 291**

If the ciphertext message produced by RSA encryption with the key  $(e, n) = (5, 2881)$  is 0504 1874 0347 0515 2088 2356 0736 0468, what is the plaintext message?