Homework Set Three: Linear Span and Bases

Directions: Submit your solutions to the Calculational Exercises and the Proof-Writing Exercises **separately** at the **beginning** of lecture on **Friday, October 19, 2007**. The two problems sets will be graded by different persons.

Calculational Exercises

1. Show that the vectors $v_1 = (1, 1, 1)$, $v_2 = (1, 2, 3)$, and $v_3 = (2, -1, 1)$ are linearly independent in $\mathbb{R}^3$. Write $v = (1, -2, 5)$ as a linear combination of v_1, v_2, and v_3.

2. Consider the complex vector space $V = \mathbb{C}^3$ and the list (v_1, v_2, v_3) of vectors in V, where
$$v_1 = (i, 0, 0), \quad v_2 = (i, 1, 0), \quad v_3 = (i, i, -1)\,.$$
   (a) Prove that $\text{span}(v_1, v_2, v_3) = V$.
   (b) Prove or disprove: (v_1, v_2, v_3) is a basis for V.

Proof-Writing Exercises

1. Let V be a vector space over $\mathbb{F}$, and suppose that $(v_1, v_2, \ldots, v_n)$ is a linearly independent list of vectors in V. Given any $w \in V$ such that
$$(v_1 + w, v_2 + w, \ldots, v_n + w)$$
is a linearly dependent list of vectors in V, prove that $w \in \text{span}(v_1, v_2, \ldots, v_n)$.

2. Let V be a finite-dimensional vector space over $\mathbb{F}$, and suppose that U is a subspace of V for which $\dim(U) = \dim(V)$. Prove that $U = V$.