MAT 115A University of California Fall 2010

Homework 2

due October 13, 2010

1. Rosen 3.1 #4, pg. 74
Use the sieve of Eratosthenes or Sage to find all primes less than 200.

2. Rosen 3.1 #5, pg. 74
Find all primes that are the difference of the fourth powers of two integers.

3. Rosen 3.1 #11, pg. 74
Let $Q_n = p_1 p_2 \cdots p_n + 1$, where $p_1, p_2, \ldots, p_n$ are the n smallest primes. Determine the smallest prime factor of Q_n for $n \leq 6$. Do you think that Q_n is prime infinitely often? (*Note:* This is an unresolved question.)

4. Rosen 3.2 #3, pg. 86
Show that there are no "prime triplets", that is, primes p, $p+2$, and $p+4$, other than 3,5, and 7.

5. Rosen 3.2 #21, pg. 87 (challenging!!)
A prime power is an integer of the form p^n, where p is prime and n is a positive integer greater than one. Find all pairs of prime powers that differ by 1. Prove that your answer is correct.

6. Rosen 3.4 #2(a,b) and #3(a,b), pg. 105
Use the Euclidean algorithm to find each of the following greatest common divisors:
(a) (51,87)
(b) (105,300)
For each pair, express the greatest common divisor of the integers as a linear combination of these integers.

7. Rosen 3.4 #19, pg. 107
Let m and n be positive integers and let a be an integer greater than 1. Show that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$.