MAT 150A University of California Fall 2011

Homework 7

due November 18, 2011 in class

Read: Artin 5.2, 5.3

1. **Artin 5.2.3** pg. 188
   Prove that **O** is not a normal subgroup of M.

2. **Artin 5.2.4** pg. 188
   Let m be an orientation-reversing motion. Prove that m^2 is a translation.

3. **Artin 5.2.5** pg. 188
   Let SM denote the subset of orientation-preserving motions of the plane. Prove that SM is a normal subgroup of M, and determine its index in M.

4. **Artin 5.2.14** pg. 189
   Find an isomorphism from the group SM to the subgroup of $GL_2(\mathbb{C})$ of matrices of the form $\left[\begin{smallmatrix} a & b \\ 0 & 1 \end{smallmatrix}\right]$, with $|a| = 1$.

5. **Artin 5.3.2** pg. 189
   List all subgroups of the group D_4, and determine which are normal.

6. **Artin 5.3.4** pg. 189
   (a) Compute the cosets of the subgroup $H = \{1, x^5\}$ in the dihedral group D_{10} explicitly.

   (b) Prove that D_{10}/H is isomorphic to D_5.

   (c) Is D_{10} isomorphic to $D_5 \times H$?