MAT 150A University of California Fall 2014

Homework 2
due October 17, 2014 in class

Read: Artin Chapters 2.3, 2.4, 2.5

1. Artin 2.1.1 (pg. 69)

   Let S be a set. Prove that the law of composition defined by $ab = a$ for all $a, b \in S$ is associative. For which sets does this law have an identity?

2. Artin 2.2.6 (pg. 70)

   Let G be a group, with multiplicative notation. We define an *opposite group* G^0 with law of composition $a \star b$ as follows: The underlying set is the same as G, but the law of composition is the opposite; that is, we define $a \star b = ba$. Prove that this defines a group.

3. Determine the elements of the cyclic group generated by $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ explicitly.

4. Artin 2.4.1 (pg. 70)

   Let a, b be elements of a group G. Assume that a has order 7 and that $a^3b = ba^3$. Prove that $ab = ba$.

5. Artin 2.2.4bcde (pg. 70)

   In which of the following cases is H a subgroup of G?

   (b) $G = \mathbb{R}^\times$ and $H = \{1, -1\}$.

   (c) $G = \mathbb{Z}^+$ and H is the set of positive integers.

   (d) $G = \mathbb{R}^\times$ and H is the set of positive reals.

   (e) $G = GL_2(\mathbb{R})$ and H is the set of all matrices $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$ with $a \neq 0$.

6. Artin 2.4.5 (pg. 70)

   Prove that every subgroup of a cyclic group is cyclic.

7. Artin 2.4.6 (pg. 70)

   (a) Let G be a cyclic group of order 6. How many of its elements generate G?

   (b) Answer the same question for cyclic groups of order 5 and 8.

   (c) Describe the number of elements that generate a cyclic group of arbitrary order n.

8. Prove that a group in which every element except the identity has order 2 is abelian.

9. Prove that the additive group $\mathbb{R}^+$ is isomorphic to the multiplicative group P of positive reals.