MAT 150A University of California Fall 2014

Homework 7

due Wednesday November 26, 2014 in class

Read: Artin 6.3, 6.4

1. Let M_n be the group of isometries in $\mathbb{R}^n$. Prove that O_n is not a normal subgroup of M_n.

2. **Artin 6.3.2** pg. 188
   Let m be an orientation-reversing motion. Prove that m^2 is a translation.

3. Let SM denote the subset of orientation-preserving motions of the plane. Prove that SM is a normal subgroup of M, and determine its index in M.

4. Find an isomorphism from the group SM to the subgroup of $GL_2(\mathbb{C})$ of matrices of the form $\left[\begin{smallmatrix} a & b \\ 0 & 1 \end{smallmatrix}\right]$, with $|a| = 1$.

5. **Artin 6.4.2(a)** pg. 188
   List all subgroups of the group D_4, and determine which ones are normal.

6. **Artin 6.4.3** pg. 188

   (a) Compute the cosets of the subgroup $H = \{1, x^5\}$ in the dihedral group D_{10} explicitly.

   (b) Prove that H is normal and that D_{10}/H is isomorphic to D_5.

   (c) Is D_{10} isomorphic to $D_5 \times H$?