

## Homework 8

due June 4, 2003 in class

1. Biggs 17.7.8 (pg. 397)
2. Biggs 17.7.12 (pg. 398)
3. Biggs 17.7.18 (pg. 398)
4. Biggs 18.3.5 (pg. 411)
5. Biggs 19.1.3 (pg. 425)
6. For integers  $0 \leq k \leq n$ , the  $q$ -binomial coefficient is defined as

$$\binom{n}{k}_q = \frac{(q)_n}{(q)_k (q)_{n-k}}$$

where  $(q)_m = (1 - q)(1 - q^2) \cdots (1 - q^m)$ . Show that  $\binom{n}{k}_q$  is the generating function of partitions with at most  $k$  parts and no part exceeding  $n - k$ .

[Hint: Show that

$$\binom{n}{k}_q = q^k \binom{n-1}{k}_q + \binom{n-1}{k-1}_q$$

and use induction.]