MAT 146 University of California Spring 2014

Homework 1

due Friday April 11, 2014 in class

1. Stanley, Chapter 1.2
Suppose that the graph G has 15 vertices and that the number of closed walks of length ℓ in G is $8^\ell + 2 \cdot 3^\ell + 3 \cdot (-1)^\ell + (-6)^\ell + 5$ for all $\ell \geq 1$. Let G' be the graph obtained from G by adding a loop at each vertex (in addition to whatever loops are already there). How many closed walks of length ℓ are there in G'?

2. Stanley, Chapter 1.4
Let $r, s \geq 1$. The complete bipartite graph $K_{r,s}$ has vertices $u_1, u_2, \ldots, u_r, v_1, \ldots, v_s$ with one edge between each u_i and v_j (so rs edges in all).

(a) By purely combinatorial reasoning, compute the number of closed walks of length ℓ in $K_{r,s}$.

(b) Deduce from (a) the eigenvalues of $K_{r,s}$.

3. Stanley, Chapter 2.4
Let G be the graph with vertex set $\mathbb{Z}_2^n$ (the same as the n-cube), and with edge set defined as follows: $\{u, v\}$ is an edge of G if u and v differ in exactly two coordinates (i.e., if $\omega(u, v) = 2$). What are the eigenvalues of G?