

## Homework 5

due Friday May 16, 2014 in class

### 1. Stanley, Chapter 5.1

1. Let  $G = \{\text{id}, \pi\}$  be a group of order two with  $\text{id}$  the identity element. Let  $G$  act on  $\{1, 2, 3, 4\}$  by  $\pi \cdot 1 = 2, \pi \cdot 2 = 1, \pi \cdot 3 = 3, \pi \cdot 4 = 4$ . Draw the Hasse diagram of the quotient poset  $B_4/G$ .
2. Do the same for the action  $\pi \cdot 1 = 2, \pi \cdot 2 = 1, \pi \cdot 3 = 4, \pi \cdot 4 = 3$ .

### 2. Stanley, Chapter 5.11

In Example 5.4(b) the Hasse diagram of  $B_5/G$  is drawn, where  $G$  is generated by the cycle  $(1, 2, 3, 4, 5)$  of order 5. Using the vertex labels shown in this figure, compute explicitly  $\hat{U}_2(12)$  and  $\hat{U}_2(13)$  as linear combinations of 123 and 124, where  $\hat{U}_2$  is defined as in the proof of Theorem 5.8. What is the matrix of  $\hat{U}_2$  with respect to the bases  $(B_5/G)_2$  and  $(B_5/G)_3$ ?

### 3. Stanley, Chapter 6.1(a)

Let  $A(m, n)$  denote the adjacency matrix (over  $\mathbb{R}$ ) of the Hasse diagram of  $L(m, n)$ . Show that if  $A(m, n)$  is nonsingular, then  $\binom{m+n}{m}$  is even.