Homework 8

due Friday June 6, 2014 in class

1. Stanley, Chapter 8.10
In how many ways can we begin with the empty partition $\emptyset$, then add $2n$ squares one at a time (always keeping a partition), then remove n squares one at a time, then add n squares one at a time, and finally remove $2n$ squares one at a time, ending up at $\emptyset$?

2. Stanley, Chapter 8.23
Let w be a *balanced* word in U and D, i.e., the same number of U's as D's. For instance, $UUDUDDDU$ is balanced. Regard U and D as linear transformations on $\mathbb{R}Y$ in the usual way. A balanced word thus takes the space $\mathbb{R}Y_n$ to itself, where Y_n is the nth level of Young's lattice Y. Show that the element $E_n = \sum_{\lambda \vdash n} f^\lambda \lambda \in \mathbb{R}Y_n$ is an eigenvector for w, and find the eigenvalue.

3. Stanley, Chapter 8.27(a)
An *increasing subsequence* of a permutation $a_1 a_2 \cdots a_n \in S_n$ is a subsequence $a_{i_1} a_{i_2} \cdots a_{i_j}$ such that $a_{i_1} < a_{i_2} < \cdots < a_{i_j}$. For instance, 2367 is an increasing subsequence of the permutation 52386417. Suppose that the permutation $w \in S_n$ is sent to a SYT of shape $\lambda = (\lambda_1, \lambda_2, \ldots)$ under the RSK algorithm. Show that λ_1 is the length of the longest increasing subsequence of w.