Homework 8

due June 6, 2014 in class

(1) For which fields F and which primes p does $x^p - x$ have a multiple root?

(2) Let F be a field of characteristic p.
   (a) Apply Proposition 15.6.7 to the polynomial $x^p + 1$.
   (b) Factor this polynomial into irreducible factors in $F[x]$.

(3) (Artin 15.7.2) Determine the irreducible polynomial of each of the elements of $\mathbb{F}_8$ in the list 15.7.8.

(4) (Artin 15.7.7) Let K be a finite field. Prove that the product of the nonzero elements of K is -1.

(5) Prove that every element of $\mathbb{F}_p$ has exactly one pth root.

(6) (Artin 15.7.8) The polynomials $f(x) = x^3+x+1$ and $g(x) = x^3+x^2+1$ are irreducible overe $\mathbb{F}_2$. Let K be the field extension obtained by adjoining a root of f, and let L be the extension obtained by adjoining of g. Describe explicitly an isomorphism from K to L, and determine the number of such isomorphisms.

(7) Determine the intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

Extra credit problem:

Use the Jordan Normal Form to prove the Spectral Theorem: every self-adjoint linear operator on a complex finite-dimensional vector space has real eigenvalues and there exists a basis with respect to which the matrix for this operator is diagonal.