

Homework 1

due January 16, 2002 in class

We will use Artin's numbering system so that "Artin 5.5.4" means Chapter 5, Section 5, Problem 4.

- (1) Artin 5.5.4 (pg. 192)
- (2) Artin 5.5.5 (pg. 193)
- (3) Artin 5.6.7 (pg. 194)
- (4) A group G acts **transitively** on a non-empty G -set S if, for all $s_1, s_2 \in S$, there exists an element $g \in G$ such that $gs_1 = s_2$. Characterize transitive G -set actions in terms of orbits. Prove your answer.
- (5) A group G acts **faithfully** on a G -set S if $gs = s$ for all $s \in S$ implies $g = 1$. Show that G acts faithfully on S if and only if no two distinct elements of G have the same action on every element of S .
- (6) Find the number of distinguishable tetrahedral dice that can be made by numbering the faces of a regular tetrahedron with one to four dots (each number of dots should occur exactly once).
- (7) Find the number of distinguishable ways the edges of a square can be painted if six colors of paint are available and the same color may be used on more than one edge.
- (8) Decide if the following statements are **true** or **false**. Briefly justify your response.
 - (a) Every G -set is also a group.
 - (b) Let S be a G -set with $s_1, s_2 \in S$ and $g \in G$. If $gs_1 = gs_2$, then $s_1 = s_2$.
 - (c) Let S be a G -set with $s \in S$ and $g_1, g_2 \in G$. If $g_1s = g_2s$, then $g_1 = g_2$.