Homework 9: last one!!!
due March 13, 2002 in class

(1) Artin 8.1.1(a) (pg. 300)
(2) Artin 8.1.5 (pg. 300)
(3) Artin 8.1.11 (pg. 301)
(4) Artin 8.2.1 (pg. 301)
(5) Artin 8.2.5 (pg. 301)
(6) Let $a = x_1 + ix_2$ and $b = x_3 + ix_4$ be two complex numbers. Show that $a\bar{a} + b\bar{b} = 1$ if and only if $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$. Conclude that there is a bijective correspondence between SU_2 and the 3-sphere S^3.
(7) Let $Q \in U_2$ and $\delta = \det Q$. Show that if ϵ is a square root of δ, then $\epsilon\bar{\epsilon} = 1$, and $\det(\bar{\epsilon}Q) = 1$.
(8) Suppose that $A, A' \in SU_2$ are trace 0 matrices corresponding to the points $y = (0, y_2, y_3, y_4)$ and $y' = (0, y_2', y_3', y_4')$ respectively. Show that if we denote the usual dot product of y and y' in matrix notation by $\langle A, A' \rangle$, then $\langle A, A' \rangle = -\frac{1}{2}\mathrm{tr}(AA')$.
(9) Let $P = \begin{pmatrix} \frac{i}{\sqrt{3}} & \frac{1+i}{\sqrt{3}} \\ \frac{-1+i}{\sqrt{3}} & \frac{-i}{\sqrt{3}} \end{pmatrix} \in SU_2$. Explicitly compute the matrix $\varphi(P)$ where $\varphi : SU_2 \to SO_3$ is the orthogonal representation.
(10) Prove the following: The set $V = \{A \in M_2(\mathbb{C}) \mid A^* = -A, \mathrm{tr}A = 0\}$ is a vector space over $\mathbb{R}$ under the usual matrix addition and

$$\mathcal{B} = \left\{ \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \right\}$$

is a basis for V.