Suggested Projects

As part of the class each registered student is required to work on one project this quarter, which should be presented at the end of the quarter in form of a talk. It is strongly encouraged that you work together in groups of 2 or 3 students. Here is a list of suggested topics. The first four topics are reading projects, where you study a chapter from a book or a research article and present the material. The last topic is an open ended problem that you can work on; your findings should also be presented at the end of the quarter. If you have your own topic in mind, please consult with me (as long as it has a connection to the class this is fine with me).

(1) **Young's Lattice and Differential Posets:** Sagan chapter 5.1
In class we will discuss the Robinson-Schensted-Knuth (RSK) algorithm which gives a correspondence between permutations and pairs of tableaux. In particular it will provide a proof of the formula

$$\sum_{\lambda \vdash n} (f^\lambda)^2 = n!,$$

where f^λ is the number of standard Young tableaux of shape λ. An alternative proof of this formula (and a generalization thereof) can be given using the notion of differential posets.

(2) **Viennot's geometric construction of the RSK correspondence:** Sagan chapter 3.6
A beautiful geometric description of the RSK correspondence is due to Viennot. This yields in particular a very elegant proof of a theorem by Schützenberger: If the permutation π corresponds to the pair of tableaux (P, Q) under RSK, then π^{-1} corresponds to the pair (Q, P).

(3) **Hall-Littlewood symmetric functions:** Macdonald chapter III 1. and 2.
There exists a family of symmetric functions $P_\lambda(x;t)$ depending on a parameter t which interpolate between the Schur functions s_λ and the monomial symmetric functions m_λ. The definition and some of their properties would be part of this presentation.

(4) **LLT symmetric functions:** A. Lascoux, B. Leclerc, J.-Y. Thibon, *Ribbon tableaux, Hall-Littlewood functions, quantum affine algebras, and unipotent varieties*, J. Math. Phys. **38** (1997) 1041–1068.
Lascoux, Leclerc and Thibon introduced a new family of symmetric functions $H_\lambda^{(k)}(x;q)$ depending on a parameter q and a positive

integer k. These LLT polynomials can be defined in terms of ribbon tableaux and are currently at the forefront of research. Mark Haiman will use the LLT polynomials in his talk on January 21 in the Discrete Mathematics seminar to give a combinatorial formula for the Macdonald polynomials.

(5) **Open ended problem:**
Consider n balls colored $1, 2, \dots, n$. A *state* is a way to place these n balls in a line of boxes indexed by $\mathbb{Z}$, under the condition that at most one ball can be placed in each box. A given state can evolve from time t to time $t+1$ according to the following rules:

(a) Every ball should be moved only once within the interval between time t and time $t+1$.
(b) Move the ball of color 1 to the nearest right empty box.
(c) Repeat (b) for the balls of color $2, 3, \dots, n$ in this order.

For example, the state

$$\dots \_\,\_\,\_\,\_\,234\,\_\,15\,\_\,\_\,\_\,\_\,\dots$$

at time t evolves to the state

$$\dots \_\,\_\,\_\,\_\,\_\,\_\,\_\,23\,\_\,145\,\_\,\dots$$

at time $t+1$. Study this system. What can you find out about it!?