MAT 149A University of California Winter 2006

Homework 6

due Wednesday February 22 in class

1. Biggs 20.7 # 2 page 272
Use the group $G_\triangle$ to provide an example of the fact that if H and K are subgroups then $H \cup K$ need not be a subgroup.

2. Biggs 20.7 # 4 page 272
Let g be a given element of a group G and let $C(g)$ denote the set of elements of G which commute with g, that is

$$C(g) = \{x \in G \mid xg = gx\}.$$

Show that $C(g)$ is a subgroup of G. What is the relationship between these subgroups and the centre $Z(G)$?

3. Biggs 20.7 # 5 page 272
If G is the group of symmetries of the square, find $C(g)$ for each g in G, and then find $Z(G)$.

4. Biggs 20.8 # 3 page 276
The symmetry group of a regular pentagon is a group of order 10. Show that it has subgroups of each of the orders allowed by Lagrange's theorem, and sketch the lattice of subgroups.

5. Biggs 21.1 # 4 page 283
List all symmetries of a regular pentagon, regarded as permutations of the corners 1,2,3,4,5, labelled in cyclic order.

6. Biggs 21.1 # 5 page 283

7. Biggs 21.2 # 2 page 284