

## Practice Midterm

1. Show that from any three integers you can choose two whose squares are congruent modulo 3.
2. Compute  $45^{47} \pmod{16}$  (without a calculator).
3. Let  $G = \{a, b, c\}$  be a set with 3 elements. Define the binary operation on  $G$  via the following table

	$a$	$b$	$c$
$a$	$c$	$a$	$b$
$b$	$a$	$b$	$c$
$c$	$b$	$c$	$a$

Is  $G$  with this binary operation a group?

4. Decide whether the statement is true or false (give a short explanation).
  - (1) There are no even permutations of order 3.
  - (2) The order of the permutation  $\beta$  is 5 and the order of the permutation  $\alpha$  is 2. Then the order of  $\alpha\beta\alpha^{-1}$  is 7.
  - (3) The permutation  $(1, 2, 3)(3, 5, 6)(4, 5, 2) \in S_6$  has order three.
  - (4) There are permutations of order 15 in  $S_8$ .
  - (5) The direct product of two cyclic groups is cyclic.