MAT 150B University of California Winter 2014

Homework 4

due February 7, 2014

1. Let $Q \in U_2(\mathbb{C})$ and $\delta = \det Q$. Show that if ϵ is a square root of δ, then $\epsilon\bar{\epsilon} = 1$, and $\det(\bar{\epsilon}Q) = 1$.

2. Suppose that A, $A' \in SU_2(\mathbb{C})$ are trace 0 matrices corresponding to the points $y = (0, y_2, y_3, y_4)$ and $y' = (0, y_2', y_3', y_4')$ respectively, under κ. Show that if we denote the usual dot product of y and y' in matrix notation by $\langle A, A'\rangle$, then $\langle A, A'\rangle = -\frac{1}{2}\operatorname{tr}(AA')$.

3. Prove the following: The set $V = \{A \in M_2(\mathbb{C}) \mid A^* = -A, \operatorname{tr} A = 0\}$ is a vector space over $\mathbb{R}$ under the usual matrix addition and

$$\mathcal{B} = \left\{ \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \right\}$$

is a basis for V.

4. Let $P = \begin{pmatrix} \frac{i}{\sqrt{3}} & \frac{1+i}{\sqrt{3}} \\ \frac{-1+i}{\sqrt{3}} & \frac{-i}{\sqrt{3}} \end{pmatrix} \in SU_2(\mathbb{C})$. Explicitly compute the matrix $\varphi(P)$ where $\varphi\colon SU_2(\mathbb{C}) \to SO_3(\mathbb{R})$ is the orthogonal representation.

5. (Artin 10.1.1) Show that the image of a representation of dimension 1 of a finite group is a cyclic group.