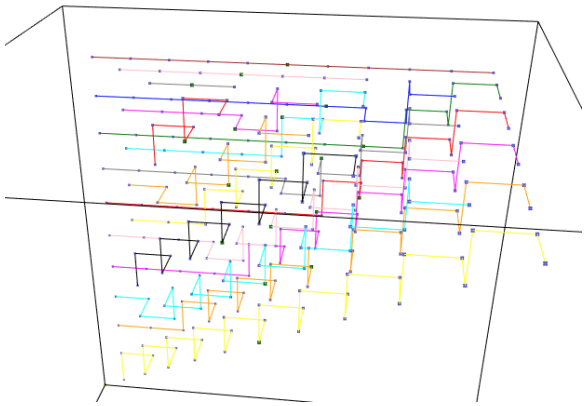
Representation Theory of Semigroups and Applications

Anne Schilling

Course Description:

The representation theory of finite groups, in particular the symmetric group, is well understood. It involves the general theory of group representations, combinatorial techniques and symmetric functions. The representation theory of finite monoids is less well known. However, in recent years it has found applications in the analysis of Markov chains, a branch of probability theory. In this course, we will study the representation theory of finite monoids and its application to Markov chains as well as the uniform block permutation algebra. The uniform block permutation algebra in turn has ties to plethysm, which involves the composition of representations. There remain many important open questions about plethysm, which will be discussed.



A list of topics includes:

1. Basics on semigroup theory (idempotents, cyclic semigroups, Green's relations, regularity, inverse monoids)
2. Representation theory of finite monoids (idempotents and simple modules, Schützenberger representation, representation theory of full transformation monoid)
3. Character theory
4. Application: Uniform block permutation algebra
5. Plethysm: history, various known results, open questions
6. Application of monoid representation theory: Markov chains

Reading:

- Benjamin Steinberg, *Representation Theory of Finite Monoids*, Springer, 2016.
- John Rhodes, Anne Schilling, *Unified theory for finite Markov chains*, Advances in Mathematics **347** (2019) 739–779.
- Rosa Orellana, Franco Saliola, Anne Schilling, Mike Zabrocki, *Plethysm and the algebra of uniform block permutations*, preprint arXiv:2112:13909.

Class Schedule: TTh 3:00-4:20pm in Welman 107

Grading: Students are asked to give a 30-minute presentation from a list of topics (or a full lecture in groups of 3 students). It is desirable to have the presentation typed up in latex format.

Prerequisites: Some knowledge of the representation theory of the symmetric group.