Convex Geometry: MATH 114

REMEMBER: *No pain no gain!* The following is a minimal list of problems to try. Some will be part of the midterms. If you have time to do more please go ahead, it is good for you!!

Exercises for first midterm

1. **First week of classes.**

   - *Section 1:* 1,3,5,6
   - *Section 2:* 5.
   - *Section 3:* 2,4
   - *Section 4:* 2.

     a) Prove that a set $S \subset \mathbb{R}^n$ is a linear subspaces if and only if it is an affine space and contains 0

     b) Prove that if two translates of an affine set intersect then the two translates are the same set.

     c) A set of $n+2$ or more points in $\mathbb{R}^n$ is affinely dependent. Prove that any maximal affinely independent set in $\mathbb{R}^n$ has exactly $n+1$.

2. **Second week of classes.**

   - *Section 5:* 2,4,6,8.
   - *Section 6:* 3,7,9,11.
   - *Section 7:*

     a) Prove that every finite set is closed.

     b) Prove that the intersection of closed sets is closed.

     c) Prove that $int(A) = int(int(A))$. (HINT: One direction is easier if you first prove that $int(A)$ is equal to the union of all open sets inside A).

     d) The *closure* of A is the intersection of all closed sets that contain A. It is denoted by $cl(A)$. Prove that a point is in $cl(A)$ if and only if for every $\delta > 0$, the open ball $B^{\circ}(x, \delta)$ contains at least one point of A.

     e) Prove or disprove: If A is open, then for any set B, $A + B$ is open.

     f) Prove: If K is convex, then $cl(K)$ is convex.

3. **Third week of classes.**

   - *Section 8:* 1,3,4,5,8,16
   - *Section 9:* 6,7,8,14.

     Prove that the Reuleaux triangle is a constant width figure.
   - *Section 10:* 1,2,5.

4. **Fourth week of classes**

- *Section 11:* 1,2,3.
- *Section 12:* 1.

  a) Use Carathéodory's theorem to show that the convex hull of a compact set is compact.

  b) In $\mathbb{R}^2$ let $x_1 = (1,0), x_2 = (1,3), x_3 = (4,3), x_4 = (4,0)$. Take $a = (7/4, 5/4)$. Show that a is in the convex hull of the x_i and then express a as a convex combination of only three of them.
- *Section 13:* 1,3.