Math 135B, Winter 2023.

Homework 2

Due: Jan. 27, 2023

1. Assume that $X_1, \ldots, X_5$ are independent random variables with the same density

$$f(x) = \begin{cases} \frac{1}{e-1} e^x & x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the moment generating function of X_1.

(b) Compute the moment generating function of $S = X_1 + \ldots + X_5$.

(c) Compute the moment generating function of $Y = X_1 + 2X_2$.

2. Assume that X is chosen at random from numbers $-1, 0, 1$, each with equal probability.

(a) Compute the moment generating function of X.

(b) Let $X_1, X_2, \ldots$ be independent and all distributed as X, and let $S_n = X_1 + \ldots + X_n$. Show that, for every $\epsilon > 0$, $P(S_n \geq \epsilon n)$ and $P(S_n \leq -\epsilon n)$ are for large n smaller that n^{-10}.

(c) Let $X_1, X_2, \ldots$ be as in (b) and let M_n be the maximal absolute value of the sum of some n consecutive terms of $X_1, \ldots, X_{n^2}$. Show that $M_n/n \to 0$ in probability.

3. (I got this problem from a high-school student. This is a harder problem, and you do not have to turn it in.) The median of a sequence of $2n+1$ numbers is the element a of the sequence such n other elements are at least a and n other elements are at most a; that is, it is the middle number after the sequence is ordered. Roll a fair die $2n+1$ times and let M_n be the median of the numbers rolled. Approximate EM_n^2 for large n and find an upper bound for the error in your approximation. (*Hints*. The distribution of M_n is symmetric. With high probability, M_n is 3 or 4. Use Problem 4 in Chapter 10.)