

### Homework 4

**Due:** Mar. 10, 2023

*Note.* You do not need to evaluate products of matrices by hand. Once you get your answer to the form in which the numerical answer can be obtained by a matrix computation, you may stop.

1. At each time  $t = 0, 1, 2, \dots$ , each of the balls  $A$  and  $B$  is either in urn 1 or urn 2. At each time step, we select ball  $A$  with probability  $p$  and ball  $B$  with probability  $1 - p$ , and then keep the selected ball in the same urn with probability  $1/3$  and place it in the other urn with probability  $2/3$ . Compute the probability that ball  $A$  is in urn 1 and ball  $B$  is in urn 2 after 6 time steps provided that: (a) initially, both balls are in urn 1; and (b) initially, each ball is independently put in one of the two urns with equal probability.

2. Assume that the Markov chain has states 1, 2, and 3 with the transition matrix

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/4 & 1/2 \\ 1/3 & 1/2 & 1/6 \end{bmatrix}.$$

Assume  $P(X_0 = 1) = P(X_0 = 2) = 1/6$ . Compute the following quantities.

(a)  $P(X_0 = X_1 = \dots = X_{10} = 3)$ .

(b)  $EX_3$  and  $EX_3^2$ .

(c)  $EX_3X_4$ .

3. Alice has two coins: coin 1 has probability 0.7 of Heads and coin 2 has probability 0.6 of Heads. She starts by tossing coin 1 twice. From then on, Alice tosses coin 1 unless her previous two tosses were both Heads, in which case she tosses coin 2. Bob knows Alice's procedure, and is in addition only told that Alice's 10th toss was Heads. Help Bob determine the (conditional) probability that Alice used coin 1 on the 10th toss.