Math 135B, Winter 2023.

Homework 8

Due: Mar. 10, 2023

Note. These three problems are adapted from job interviews on Wall Street. Assume that you may use a computer during the interview for coding (but not for looking for a solution on the internet). These seem difficult, albeit not impossible, problems to solve completely under pressure, so I suspect that the interviewers mainly want to gauge the candidate's reaction in such a situation. To get a bit of experience on how you might react, you are required, as a part of your submitted work, to write down an answer to each of the three problems *before you see the solution*. Each of your answers needs to at least outline an approach in a coherent language, and needs to contain a numerical answer, which can be just a guess if you are unable to make much progress; imagine that you are addressing your response to an interviewer. After that, you may verify and complete your work by consulting the solutions. (Of course, there is no penalty for incorrect preliminary answers.)

1. Alice has a coin with probability 0.6 of Heads, and Bob has coin with probability 0.3 of Heads. They toss a coin repeatedly: if the number of Heads tossed so far is odd, Alice tosses her coin, while if the number of Heads tossed so far is even, Bob tosses his coin. Let p_n be the probability of even number of Heads after n tosses. Approximate p_{1000}.

2. Two tokens are both initially positioned at 0 and move on nonnegative integers $0, 1, \ldots$ as follows. There are two coins: the red coin is fair, with Heads probability $1/2$, while the blue coin has Heads probability $2/3$. Each minute, exactly one of the token makes a move based on the outcome of a toss of one of the coins: if the toss is Heads, the move is two unit steps to the right, and if the toss is Tails, the move is one unit step to the right. Here are the rules on which coin is used and which token makes the move:

- if both token occupy the same position, the red coin is used for the toss, and one of the tokens makes the move and the other stays put;
- otherwise, the blue coin is used for the toss, and the token that is behind (i.e., to the left) makes the move and the token that is ahead stays put.

Let p_n be the probability that the two tokens ever both simultaneously occupy n. Approximate p_{1000}.

3. A casino offers the following card game. A standard deck is shuffled and the dealer draws cards one by one, without replacement. You may ask the dealer to stop at any time. For each red card drawn, you win \$1; and for each black card drawn, you lose \$1. How much are you willing to pay to play the game?