

Final Review

Winter 2008, Math 16A , A.I. David Haws

This is meant as an aid. It does *not* cover everything!

1) Find x such that the distance between $(2, -1)$ and $(x, 2)$ is 5.

2) Complete the square to write the equation of the circle in standard form. What is the center and the radius of the circle?

$$4x^2 + 4y^2 - 4x + 2y - 1 = 0.$$

3) Find all points of intersection (x, y) for the functions

$$y = 5 + x \quad y = -x^2 + 7$$

4) Find all values of k for which the graphs of $y = x^2 + 7$ and $y = kx^2 + 3$ do **not** intersect.

5) a) Find the domain of the function $h(x) = \sqrt{(x+4)(x-1)^2(x-2)(x-7)}$

b) What is the domain of the function $j(x) = \frac{1}{h(x)}$?

6) Find the domain and range of $f(x) = \frac{5}{\sqrt{x-3}}$.

7) a) Find the inverse function of $f(x) = x^3 + 1$.

b) Prove that your answer in a) is the inverse by showing $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

8) Find the limit (if it exists)

a) $\lim_{x \rightarrow 0} \frac{x^2+1}{3x-2}$

b) $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$

c) $\lim_{x \rightarrow 3} \frac{x^2+x-12}{x-3}$

d) $\lim_{x \rightarrow \infty} \frac{2x-1}{3x+7}$

e) $\lim_{x \rightarrow -\infty} \frac{2x^2}{x-1} - \frac{3x}{x+1}$

f) $\lim_{x \rightarrow \infty} \frac{x^2+2x-1}{3x^4-x^2+2x-1}$

g) $\lim_{x \rightarrow 2} \frac{4-\frac{8}{x}}{x-2}$

9) Find $\lim_{x \rightarrow -2^-} g(x)$, $\lim_{x \rightarrow -2^+} g(x)$, $\lim_{x \rightarrow 2^-} g(x)$, and $\lim_{x \rightarrow 2^+} g(x)$. If they exist find $\lim_{x \rightarrow -2} g(x)$ and $\lim_{x \rightarrow 2} g(x)$.

$$g(x) = \begin{cases} 2 & x < 2 \\ \sqrt{4-x^2} & -2 \leq x \leq 2 \\ x-2 & x > 2 \end{cases}$$

10) a) Is $f(x)$ continuous at $x = -1$ or $x = 1$?

$$f(x) = \begin{cases} x+1 & x < -1 \\ -\sqrt{1-x^2} & -1 \leq x \leq 1 \\ 2-x & x > 1 \end{cases}$$

b) Is $f(x)$ continuous at $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$?

$$f(x) = \begin{cases} 2 \cos(x) & 0 \leq x \leq \frac{\pi}{2} \\ -\sin(x) & \frac{\pi}{2} < x \leq \frac{3\pi}{2} \\ 1 & \frac{3\pi}{2} < x \leq 2\pi \end{cases}$$

11) Find an a so that the following function is continuous

$$f(x) = \begin{cases} ax^2 + 2x + 5 & 0 \leq x \leq 1 \\ 9x & x > 1 \end{cases}$$

12) Use the definition of the derivative to find $f'(x)$.

a) $f(x) = \frac{x+5}{x}$ $f(x) = 1 - x^2$

13) Find two values of θ , $0 \leq \theta \leq 2\pi$ such that

a) $\tan \theta = -1$ b) $\cot \theta = -\sqrt{3}$

14) A rocket travels upward given by a height function $s(t) = 8t^3 + 24$.

a) Find the average rate of change of $s(t)$ from $[2, 4]$.

b) Find the velocity when $t = 2$ and $t = 3$.

c) Find the acceleration when $t = 5$.

15) Find $\frac{dy}{dx} = y'$. Do **not** simplify

a) $y = x^2\sqrt[4]{x^3+1}$

b) $y = \tan^5(7x^3+5)$

c) $y = \frac{(2x^4+1)^3}{(6x^3+\sqrt{x})^4}$

d) $y = (\sin 8x)(\cot 10x)$

e) $y = (6x^5 - 4x^2 + 2)^{50}$

b) $y = \sin\left(\frac{x+2}{x-1}\right)$

16) a) Find the equation of the tangent line to $f(x) = x(x^2 + \frac{1}{x})$ at $(1, 2)$.

b) Find the line perpendicular to the one in part a).

17) Find $f''(x)$.

a) $f(x) = \sqrt{x^3+2}$

b) $f(x) = \sin(x^2)$

18) Find $\frac{dy}{dx} = y'$.

a) $y^3 + \cos y + x^2 = 2$

b) $y^3 + xy + x^2 = 3$

19) Find an equation of the line tangent to the graph of $y + \tan y = x^3 + x + \pi$ at the point $(0, \pi)$.

20) Find the absolute extrema of the function $f(x) = x^3 + 2x^2 - 3x + 4$ on $[-3, 2]$.

21) Find all critical points of $f(x) = \frac{-x+2}{x+2}$. Do not classify extrema.

22) An airplane flying at an altitude of 6 miles passes directly over a radar antenna. When the airplane is 10 miles directly away from the radar tower (call it s), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the airplane?

23) An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter running track. Find the dimensions that will make the area of the rectangular region as large as possible.

24) Use differentials to approximate.

a) $\sqrt{101}$

b) $\tan(43^\circ)$

c) $\sin 31^\circ$

d) $\sqrt[3]{63}$

a) $\frac{1}{(26)^{\frac{2}{3}}}$

25) For the following, find all relative extrema, points of inflection, intercepts, asymptotes and sketch the graph.

a) $y = x(x-4)^3$

b) $y = \frac{x^2-6x+12}{x-4}$