MAT 127B-01 Winter 05 Midterm 2

1.(20 pts) Let

$$f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

a. Is f continuous at $x = 0$? Explain.
b. Is f differentiable at $x \neq 0$. Explain your answer and compute $f'(x)$ if possible.
c. Is f differentiable at $x = 0$. Explain your answer and compute $f'(x)$ if possible.

2.(20 pts) Suppose that f is differentiable and $2 \leq f'(x) \leq 3$ on $\mathbb{R}$. Show that for $x \geq 0$

$$2x \leq f(x) - f(0) \leq 3x$$

3.(20 pts) Find the following limits if they exist.

$$\lim_{x \to 0} \frac{\sin x - x \cos x}{x - \sin x}$$

$$\lim_{x \to 1} x^{\frac{1}{1-x}}$$

4.(20 pts)
a. Suppose $f(x) = \sqrt{1+x}$ show that

$$f^{(k)}(x) = \frac{(-1)^{k-1}}{2^k} 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2k-3)(1+x)^{\frac{1-2k}{2}}$$

b. Find the Taylor series of $f(x)$ where the remainder is of degree n.
c. What is the remainder $R_n(x)$.

5.(20 pts) Assume f and f' are differentiable and f'' is continuous on $\mathbb{R}$. Use L'Hospital's rule to show

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Justify your steps.