MAT 67, Homework 5, due 2/11/08

1. Is the following set of matrices a basis for $\mathbb{R}^{2\times 2}$?

$$\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

2. What is the dimension of $\mathbb{R}^{m\times n}$? (To find this, write down a set of vectors that you think is a basis for this vector space. You needn't prove that it's a basis.)

3. What is the dimension of the subspace of $\mathbb{R}^{m\times n}$ spanned by the following set of vectors?

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

4. Find a set of vectors that is a basis for $\mathbb{P}_n(\mathbb{F})$, the vector space of polynomials of degree at most n over $\mathbb{F}$. (Show that the set of vectors is is linearly independent and that it spans $\mathbb{P}_n(\mathbb{F})$).

5. What is the dimension of $\mathbb{P}_n(\mathbb{F})$, the vector space of polynomials of degree at most n over $\mathbb{F}$? (Refer to #4.)

6. Exercises 2, 4, 7 on page 61

7. Which of the following maps are linear? (Proof or counterexample.)

   $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (2x + y, -3)$.

   $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (2x + y, -3y)$.

   $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (2x + y, y^3)$.

   $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (sinx, cosy)$.

8. Find the nullspaces and ranges of the following linear maps.

   $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (2x + y, 5y)$.

   $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (y, 5y)$.

$T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (0, x)$.

$T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (0, 0)$.