

MAT 67, Homework 7, due 2/25/08

1. Solve the following systems of linear equations by using Gaussian elimination.

(a)

$$\begin{aligned}x_1 + 2x_2 - 2x_3 + 3x_4 &= 2 \\2x_1 + 4x_2 - 3x_3 + 4x_4 &= 5 \\5x_1 + 10x_2 - 8x_3 + 11x_4 &= 12\end{aligned}$$

(b)

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= 4 \\x_1 + 3x_2 + x_3 &= 11 \\2x_1 + 5x_2 - 4x_3 &= 13 \\2x_1 + 6x_2 + 2x_3 &= 22\end{aligned}$$

(c)

$$\begin{aligned}x_1 + 2x_2 - 2x_3 + 3x_4 &= 2 \\2x_1 + 4x_2 - 3x_3 + 4x_4 &= 3 \\x_1 + 10x_2 - 8x_3 + 11x_4 &= 12\end{aligned}$$

2. Find the inverses of the following matrices using Gaussian elimination.

(a)

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -1 & 1 & -3 \\ 2 & 1 & 1 \\ 2 & 5 & -4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 1 & 3 & 1 & 5 \\ 2 & 5 & -4 & 2 \\ 1 & 3 & 1 & 3 \end{bmatrix}$$

3. Factor the matrices in the preceding exercise into elementary matrices.
4. Find the null spaces and ranges of the linear maps $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by the following matrices:

(a)

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -1 & 1 & -3 \\ 2 & -2 & 6 \\ 1 & -1 & 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \end{bmatrix}$$

5. Prove that performing a row operation has the same effect as left multiplication by the corresponding elementary matrix.
6. Proof writing exercises 4, 6, 7 on page 80-81