

MATH 21B. Final Exam (Sample).

June 4, 2008

The exam is “closed book, closed notes, no calculators”. Show all the work: Correct answer without justification can result in a very low score on the problem.

1. Evaluate the integral

$$\int \frac{1}{x^2} \exp\left(\frac{x-1}{x}\right) dx.$$

Hint: Use a substitution.

2. State the formula for the radioactive decay. Explain the terms which appear in the formula. Explain what is the half-life of the radioactive material and how to compute it using the radioactive decay formula.

3. Compute the length of the parameterized curve

$$x = e^t - t, y = 4e^{t/2}, -1 \leq t \leq 1.$$

4. Compute the volume of the solid of revolution obtained by rotating (about the line $x = 0$) the region bounded by the curve

$$y = 2x(1 - x)$$

and the x -axis. Use any method you like.

5. State and prove the integration by parts formula.

6. You have a vertical conical tank of height 10 ft and the radius 4 ft filled with a liquid to within 2 ft from the rim. The density of the liquid is $\delta = 100\text{lb/ft}^3$. Compute the total work needed to pump all liquid from the tank.

7. Compute the improper integral

$$\int_0^3 \frac{dx}{x-1}.$$

Solution: See the textbook, page 610.

8. Using the trapezoid rule estimate the integral

$$\int_1^2 x dx$$

where you subdivide the interval in 4 equal parts.

9. Replace the following equation by equivalent Cartesian equation and identify the graph:

$$r \cos(\theta) = -4$$

Draw a picture of the graph.

Solution. Textbook, page 701.

10. Compute the integral

$$\int \frac{x^2 dx}{\sqrt{9-x^2}}.$$

Solution. Textbook, page 572.