ESP
Kouba
Worksheet 12

1. Find an equation of the line tangent to the graph of $y = 2x \cos 3x$ at $x = \pi/2$.

2. Consider the line $f(x) = 2x + 3$.

    a.   Sketch the graph of f.

    b.   The point $(0, 3)$ lies on the graph of f. Find all points (x, y) which lie on the graph of f at a distance of 5 from the point $(0, 3)$.

3. A baseball is thrown upward from the top of a 64-foot high building with a velocity of 48 ft./sec.

    a.   How long does it take the ball to reach its highest point ?

    b.   How high above the ground will it go ?

    c.   After how many seconds will it hit the ground ?

    d.   What is the ball's velocity as it strikes the ground ?

4. The function $y = Ax^3 + Bx^2 + Cx + D$ has a relative maximum value at the point $(-1, 2)$ and a relative minimum value at the point $(1, -1)$. Determine the value of constants A, B, C, and D.

5. Assume that function f is continuous on the interval [a, b] and differentiable on (a, b) with $f'(x) < 0$ for $a < x < b$.

    a.   Draw a sketch illustrating that these assumptions imply that $f(b) < f(a)$.

    b.   Write a short proof that $f(b) < f(a)$.

6. Find the slope of the line normal to the graph of $y = (1 + \sin 8x)^5$ at $x = \pi/24$.

7. For what values of x is $f'(x) = 0$ for $f(x) = 2x - \tan x$?

8. Assume that function f is continuous on the interval $[0, 3]$ and differentiable on $(0, 3)$. Show that there is some number c in $[0, 3]$ satisfying

$$f'(c) = f(2) - f(1) .$$

9. Assume that the number N of animals at time t (years) in an isolated herd of elk in Yellowstone National Park is modeled by the equation

$$N(t) = \frac{9990}{t^2 + 27} .$$

   a. How many elk are there initially ? at time $t = 10$ years ?
   b. What is the average rate of change in the number of elk during the period from $t = 1$ year to $t = 5$ years ?
   c. At what rate is the number of elk changing when $t = 1$ year ? $t = 5$ years ?
   d. For what value of t is the number of elk changing most rapidly ?

10. Build a rectangular corral using 400 feet of fencing. What length and width will result in the corral of largest possible area ?

11. You are to construct an open rectangular box with a square base using 12 ft.^{2} of material. What should the dimensions of the box be in order that the volume of the box be as large as possible ?

12. Determine which point(s) (x, y) on the graph of $x = \sqrt{y}$ are nearest the point $(0, 3)$.

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13. Each day a 100-room hotel is filled to capacity at $50 per room. For each increase of $5 per room, the manager has estimated that 3 fewer rooms are rented. What price should be charged per room in order that the hotel's daily revenue be as large as possible ?

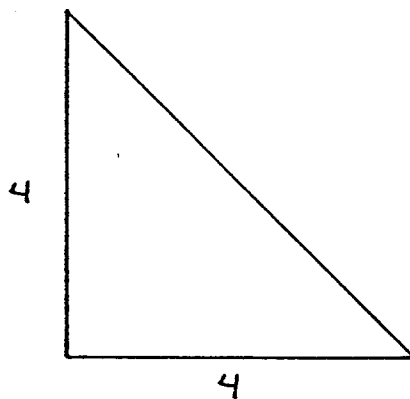14. a. Determine the roots of the following quadratic functions.

$$i. \quad f(x) = x^2 - x - 6$$
$$ii. \quad f(x) = 2x^2 - 3x + 1$$

b. Assume that r and s are roots of a quadratic function f. Prove that

$$f'(r) + f'(s) = 0.$$

15. What is the radius of the largest sphere which will " slip through " the given isosceles right triangle ?

16. A heavy rock is dropped from a suspension bridge into the icy water below. A big splash is heard. If the total time elapsed from the moment the rock is released until the moment the splash is heard is 10 seconds, how high is the bridge above the water ? *Assume that the speed of sound is 1115 ft./sec.*

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