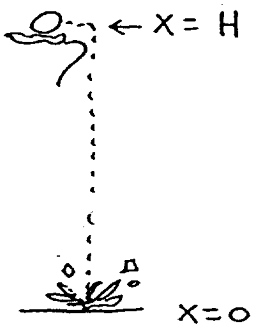
ESP
Kouba
Worksheet 13 Solutions

1.) $f'(x) = 2\sec 2x \cdot \tan 2x = 0 \rightarrow \sec 2x = 0$ or $\tan 2x = 0$
$\rightarrow 2x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \cdots \rightarrow x = 0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \cdots$

2.)



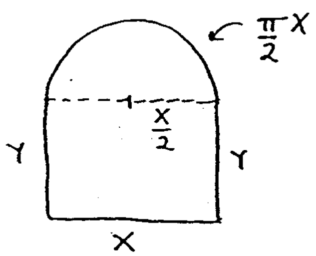
$x''(t) = -32$ H: height
$x'(t) = -32t$ T: falling time
$x(t) = -16t^2 + H$

60 mph = 88 ft./sec.

$x(T) = 0 \rightarrow \boxed{0 = -16T^2 + H}$ and
$x'(T) = -88 \rightarrow \boxed{-32T = -88} \rightarrow$

$T = 2.75$ sec $\rightarrow H = 121$ feet.

3.)



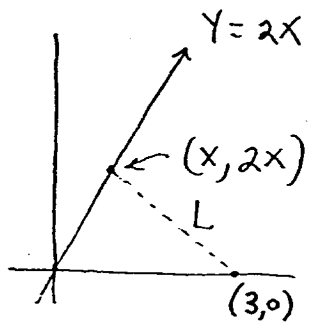
$2Y + X + \frac{\pi}{2}X = 10 \rightarrow$
$Y = 5 - \frac{1}{2}X - \frac{\pi}{4}X$ and
maximize total area
$A = XY + \frac{1}{2}\pi\left(\frac{X}{2}\right)^2 \rightarrow$

$A = x\left(5 - \frac{1}{2}x - \frac{\pi}{4}x\right) + \frac{\pi}{8}x^2 = 5x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2 \rightarrow$
$A = 5x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2 \rightarrow$
$A' = 5 - x - \frac{\pi}{4}x = 0 \rightarrow x = \frac{5}{1+\frac{\pi}{4}} = \frac{20}{4+\pi}$

A': $+$ 0 $-$

$x = \frac{20}{4+\pi} \approx 2.8$ ft, $y = \frac{10}{4+\pi} \approx 1.4$ ft, max. $A \approx 7$ ft.^{2}

4.)



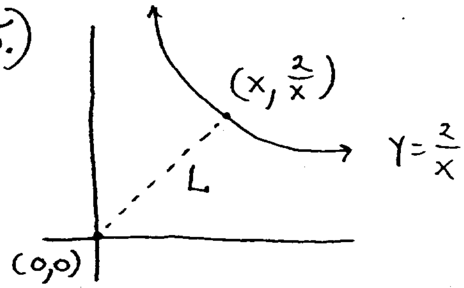
minimize distance
$L = \sqrt{(2x-0)^2 + (x-3)^2}$
$= \sqrt{4x^2 + (x-3)^2} \rightarrow$

$L' = \frac{1}{2}\left(4x^2 + (x-3)^2\right)^{-1/2} \cdot \{8x + 2(x-3)\} = \frac{10x-6}{2\sqrt{4x^2+(x-3)^2}} = 0 \rightarrow$
$10x - 6 = 0 \rightarrow x = 3/5$

L' $\quad - \quad 0 \quad +$

$x = 3/5$, $y = 6/5$, min. $L = \sqrt{\frac{180}{25}} = \frac{6\sqrt{5}}{5}$

5.)



minimize distance

$$L = \sqrt{\left(\frac{2}{x} - 0\right)^2 + (x-0)^2}$$

$$= \sqrt{\frac{4}{x^2} + x^2} \rightarrow$$

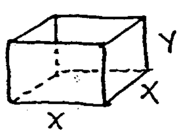
$$L' = \frac{1}{2}\left(\frac{4}{x^2} + x^2\right)^{-1/2} \cdot \left\{\frac{-8}{x^3} + 2x\right\} = \frac{2x^4 - 8}{2x^3\sqrt{\frac{4}{x^2} + x^2}} = 0 \rightarrow$$

$$2x^4 - 8 = 0 \rightarrow x^4 = 4 \rightarrow x = 4^{1/4}$$

L' $\quad 0$

$x = 4^{1/4} \approx 1.4$, $y = \frac{2}{4^{1/4}} \approx 1.4$, min. $L = 2$

6.)



$$x^2 y = 4 \rightarrow y = \frac{4}{x^2}$$

and minimize surface area

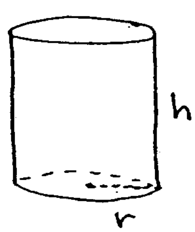
$$A = x^2 + 4xy = x^2 + 4x \cdot \frac{4}{x^2} = x^2 + \frac{16}{x} \rightarrow$$

$$A' = 2x - \frac{16}{x^2} = \frac{2x^3 - 16}{x^2} = 0 \rightarrow 2x^3 - 16 = 0 \rightarrow x = 2$$

A' $\quad - \quad 0 \quad +$

$x = 2$ ft., $y = 1$ ft., min. $A = 12$ ft.^{2}

7.)



$$\pi r^2 + 2\pi r h = 3\pi \rightarrow$$

$$h = \frac{3\pi - \pi r^2}{2\pi r} = \frac{3 - r^2}{2r}$$

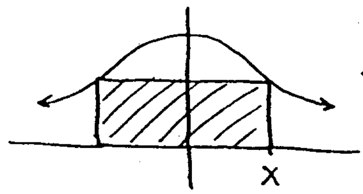
and maximize volume

$$V = \pi r^2 h = \pi r^2 \cdot \frac{3 - r^2}{2r} = \frac{3}{2}\pi r - \frac{\pi}{2} r^3 \rightarrow$$

$$V' = \frac{3}{2}\pi - \frac{3}{2}\pi r^2 = \frac{3}{2}\pi(1 - r^2) = 0 \rightarrow r = 1$$

V' $+ \quad 0 \quad -$

$r = 1$ m., $h = 1$ m., max. $V = \pi$ m.^{3}

8.) 

$y = \dfrac{4}{x^2+1}$ Maximize area of rectangle

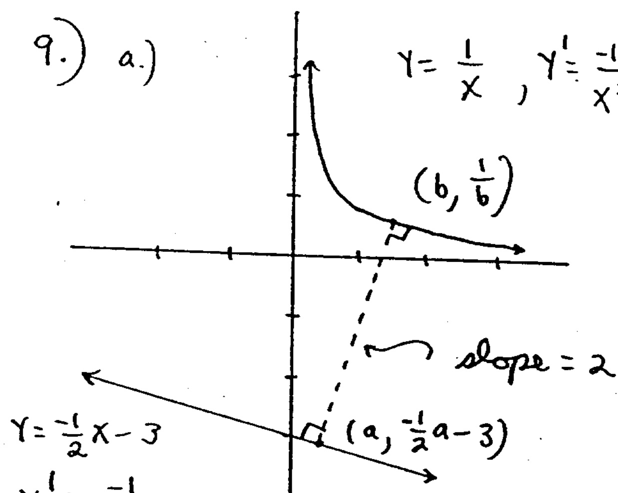
$$A = 2x \cdot y = 2x \cdot \frac{4}{x^2+1} = \frac{8x}{x^2+1} \rightarrow$$

$$A' = \frac{(x^2+1)(8) - 8x \cdot (2x)}{(x^2+1)^2} = \frac{8-8x^2}{(x^2+1)^2} = \frac{8(1-x^2)}{(x^2+1)^2} = 0 \rightarrow$$

$8(1-x^2) = 0 \rightarrow x = 1$ $+ \quad 0 \quad -$ A'

so rectangle is 2 by 2 $x = 1$, $y = 2$

max. $A = 4$

9.) a.) 

$Y' = \dfrac{-1}{2}$

$Y = \dfrac{1}{x}$, $Y' = \dfrac{-1}{x^2}$ b.) Determine a and b so that dotted line has slope 2 :

$b^2 = 2$ and

$$\frac{\frac{1}{b} - \left(\frac{-1}{2}a - 3\right)}{b-a} = 2 \rightarrow$$

$b = \sqrt{2}$ and

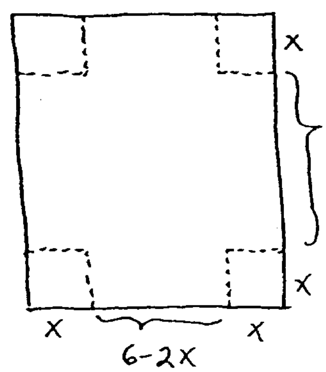
$$\frac{1}{\sqrt{2}} + \frac{1}{2}a + 3 = 2(\sqrt{2} - a) \rightarrow a = \frac{2}{5}\left(\frac{3}{\sqrt{2}} - 3\right), \text{ so}$$

$\left(b, \frac{1}{b}\right) = \left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(a, \frac{-1}{2}a - 3\right) = (-.35, -2.82)$ and

distance between graphs is

$$L = \sqrt{\left(-2.82 - \frac{1}{\sqrt{2}}\right)^2 + \left(-.35 - \sqrt{2}\right)^2} \approx 3.94$$

10.)



maximize volume

$$V = x(6-2x)(10-2x)$$
$$= (6x-2x^2)(10-2x) \longrightarrow$$

$$V' = (6x-2x^2)(-2) + (6-4x)(10-2x)$$

$$= -12x + 4x^2 + 60 - 12x - 40x + 8x^2$$
$$= 12x^2 - 64x + 60$$
$$= 4(3x^2 - 16x + 15) = 0 \longrightarrow$$

$$x = \frac{16 \pm \sqrt{16^2 - 12(15)}}{6} = \frac{16 \pm 2\sqrt{19}}{6} = \frac{8 \pm \sqrt{19}}{3} \longrightarrow$$

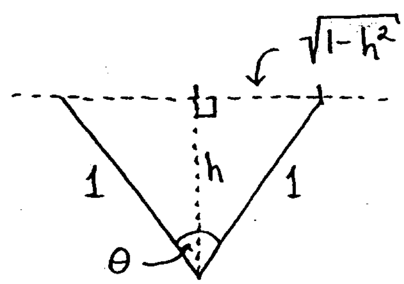
$x = 4.12$ ft. or $\boxed{x = 1.21 \text{ ft.}}$

impossible

V' $+$ 0 $-$

$x = 1.21$ ft., max. $V = 32.8$ ft.^{3}

11.)



Maximize flow through gutter $\longrightarrow$ maximize area of triangle $\longrightarrow$

$$A = h\sqrt{1-h^2} \longrightarrow A' = h \cdot \tfrac{1}{2}(1-h^2)^{-\frac{1}{2}}(-2h) + \sqrt{1-h^2}$$

$$= \frac{-h^2}{\sqrt{1-h^2}} + \frac{\sqrt{1-h^2}}{1} = \frac{1-2h^2}{\sqrt{1-h^2}} = 0 \rightarrow h = \frac{1}{\sqrt{2}} \longrightarrow \theta = \frac{\pi}{2}$$

A' 0

$h = \frac{1}{\sqrt{2}}$ ft. ≈ 0.71 ft., $\theta = \frac{\pi}{2}$, and max. $A = \frac{1}{2}$ ft.^{2}