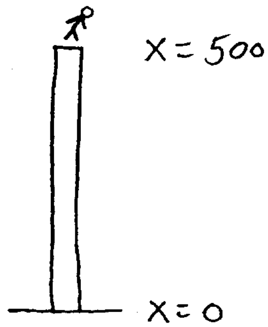
ESP
Kouba
Worksheet 15 Solutions

1.)



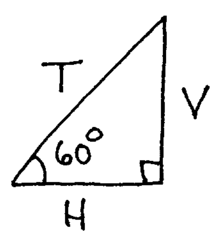
$x''(t) = -32 \rightarrow$
$x'(t) = -32t + c$
$x'(0) = -60$ mph $= -88$ ft./sec. $\rightarrow$
$\boxed{x'(t) = -32t - 88} \rightarrow$
$x(t) = -16t^2 - 88t + c$
$x(0) = 500 \rightarrow$
$\boxed{x(t) = -16t^2 - 88t + 500}$;

to hit ground : $x(t) = 0 \rightarrow -16t^2 - 88t + 500 = 0 \rightarrow$

$t = \dfrac{88 \pm \sqrt{39744}}{-32} = 3.48$ sec. and

$x'(3.48) = -199.36$ ft./sec. $= -135.9$ mph

2.)



Let T be initial velocity of rocket and
V : "vertical component" of T
H : "horizontal component" of T ;

analyze vertical motion of rocket :
acc. $s''(t) = -32$ ft./sec.^{2} $\rightarrow$
$s'(t) = -32t + c$ (Since rocket travels for 15 seconds, it will be at its highest point in 7.5 seconds, i.e. $s'(7.5) = 0$!
$\rightarrow 0 = -32(7.5) + c \rightarrow c = 240) \rightarrow$
vel. $\underline{s'(t) = 240 - 32t}$; then

$V = s'(0) = 240$ ft./sec and

$\sin 60° = V/T \rightarrow T = \dfrac{V}{\sin 60°} = \dfrac{240}{\sqrt{3}/2} \rightarrow$

initial velocity $T \approx 277.13$ ft./sec. ;

$\cos 60° = \dfrac{H}{T} \rightarrow H = T\cos 60° \approx (277.13)(\tfrac{1}{2}) \approx 138.6$ ft./s.

so rocket travels $(138.6)(15) = 2078.46$ ft. ;

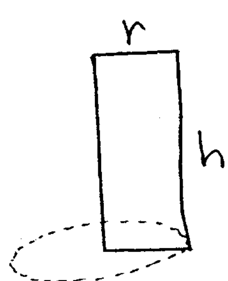
since $s'(t) = 240 - 32t \rightarrow s(t) = 240t - 16t^2 + c$

$(s(0) = 0 \rightarrow 0 = 0 + c \rightarrow c = 0) \rightarrow$ ht. $s(t) = 240t - 16t^2$

and so maximum height is

$s(7.5) = 240(7.5) - 16(7.5)^2 = 900$ ft. .

3.)



$2r + 2h = 24$ in. $\rightarrow h = 12 - r$

maximize volume

$V = \pi r^2 h = \pi r^2(12 - r)$

$= 12\pi r^2 - \pi r^3 \rightarrow$

$V' = 24\pi r - 3\pi r^2 = 3\pi r(8 - r) = 0 \rightarrow r = 8$ in. $h = 4$ in.

$\begin{array}{c} + \;\; 0 \;\; - \\ \hline r = 8\text{ in.} \end{array} \; V'$

max.

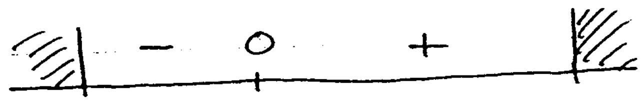
$V = 256\pi$ in.^{3}

4.) $X + Y = 1 \rightarrow Y = 1 - X$ and let

$S = X^2 + Y^3 = X^2 + (1-X)^3 \rightarrow S' = 2X - 3(1-X)^2$

$$= -3x^2 + 8x - 3 = 0 \rightarrow x = \frac{-8 \pm \sqrt{64-36}}{-6} = \frac{-8 \pm 2\sqrt{7}}{-6}$$

$$= \cancel{2.21} \text{ or } .45 \text{ and } y = .55 \rightarrow$$



$x=0$, $y=1$ ↑ max. $S=1$

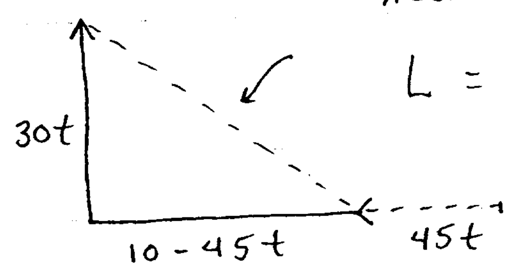
$x=.45$, $y=.55$ ↑ min. $S=0.369$

$x=1$, $y=0$ ↑ max $S=1$

a.) max : #'s are 0 and 1

b.) min : #'s are .45 and .55 .

5.)



minimize length

$$L = \sqrt{(30t)^2 + (10-45t)^2} \rightarrow$$

$$L' = \tfrac{1}{2}(\sim)^{-\frac{1}{2}} \cdot \{1800t + 2(10-45t)(-45)\} = 0 \rightarrow$$

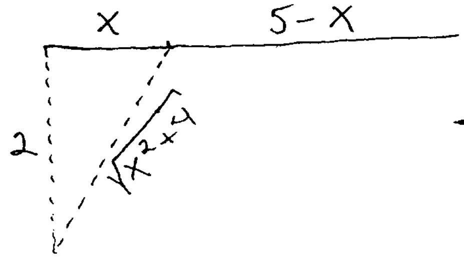
$$5850t - 900 = 0 \rightarrow t = .1538 \text{ hr. and}$$

$$L = \sqrt{30.77} = 5.55 \text{ miles}.$$

$-$ 0 $+$ L'

$t = 0.1538$ hr.

6.)



minimize time

$$T = T_{swim} + T_{jog}$$

$$= \frac{\sqrt{x^2+4}}{2} + \frac{5-x}{4} \rightarrow$$

$$T' = \frac{1}{2} \cdot \frac{1}{\cancel{2}} (x^2+4)^{-1/2} \cdot \cancel{2}x - \frac{1}{4} = \frac{x}{2\sqrt{x^2+4}} - \frac{1}{4} = 0 \rightarrow$$

$$4x = 2\sqrt{x^2+4} \rightarrow 4x^2 = x^2+4 \rightarrow x^2 = 4/3 \rightarrow$$

$x = \frac{2}{\sqrt{3}}$ miles. $\underset{x = 2/\sqrt{3} \text{ mi}}{\overset{0}{\text{———+———}}}$ T'

min. $T \approx 2.12$ hr.

7.) $Y'' = 1 - x^2 \rightarrow$

$Y' = x - \frac{1}{3}x^3 + c$ and $Y'(1) = -1$ (slope)

$-1 = 1 - \frac{1}{3} + c \rightarrow c = -5/3 \rightarrow$

$Y' = x - \frac{1}{3}x^3 - 5/3 \rightarrow$

$Y = \frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{5}{3}x + c$ and $Y(1) = 1$

$1 = \frac{1}{2} - \frac{1}{12} - \frac{5}{3} + c \rightarrow c = \frac{9}{4} \rightarrow$

$Y = \frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{5}{3}x + \frac{9}{4}$.