ESP
Kouba
Worksheet 16

1. Evaluate the following limits.

a. $\lim_{t \to 0} (1+t)^{1/t}$

b. $\lim_{n \to +\infty} (1+1/n)^{n}$

c. $\lim_{n \to -\infty} (1+1/n)^{n}$

d. $\lim_{n \to +\infty} (1-1/n)^{n}$

e. $\lim_{n \to +\infty} (1+3/n)^{4n}$

f. $\lim_{n \to +\infty} (1+1/n^{2})^{n}$

g. $\lim_{n \to +\infty} (1-2/n)^{n^2}$

h. $\lim_{t \to 0^+} (1+t)^{1/\sqrt{t}}$

i. $\lim_{n \to +\infty} \left(\frac{n+2}{n}\right)^{3n}$

j. $\lim_{n \to -\infty} \left(\frac{n}{n-1}\right)^{n^2}$

2. Solve for x.

a. $3^x = 27$

b. $3^x = 20$

c. $(1+x/12)^{36} = 200$

d. $\ln x = 10$

e. $\ln 2x = \ln\sqrt{x^2+1}$

f. $\ln x + \ln(x-3) = \ln 4$

g. $\ln(2x+1) - \ln(x-3) = \ln x$

3. Assume that y is a function of x and find $y' = dy/dx$.

a. $x^2 + y = 1$

b. $x^2 + y^2 = 1$

c. $xy^3 + 5y = x$

d. $(x+4y)^5 = 7$

e. $x\tan y + \sin 3x = \sin y^3$

f. $x^2\sec^2 y^2 = y - x$

g. $\cot(x - \sin\sqrt{y}) = y/x$

4. Determine an equation for the line tangent to the graph of $y^3 - x^2 + 1 = 0$ at $x = 3$.

5. Determine an equation of the line normal to the graph of $y^3 = xy^2$ at the point $(8^{1/4}, 2)$.

6. Assume that y is a function of x and determine $y' = dy/dx$.

a. $y = x^5$

b. $y^5 = x$

c. $\cos(xy) = \tan(x - y)$

d. $x^2 y + x y^2 = 1$

e. $\ln y = x \ln x$

f. $y = x^x$

g. $y = 3^x$

h. $y = x \cos x^3$

i. $y = x^y$

j. $\sqrt{5x} = x\, y^2$

k. $y = 2^{2^2}$

l. $y = x^{x^x}$

m. $y = 7^{\ln x}$

n. $y = x \operatorname{arcsec} x$

o. $y = \arcsin^2 \sqrt{x}$

p. $y = (\arccos 3/x)^5$

q. $x = \arctan y$

r. $y = \arctan y$

7. With each of the following functions are given numbers x_1 (the initial x-value) and x_2 (the final x-value). Compute the associated exact change in functional value, Δf, and the differential of f (approximate change in functional value), $d f$.

a. $f(x) = x^3 + x - 1$
   - i. $x_1 = 1,\ x_2 = 4$
   - ii. $x_1 = 1,\ x_2 = 2$
   - ii. $x_1 = 1,\ x_2 = 1.1$
   - iv. $x_1 = 1,\ x_2 = 1.01$

b. $f(x) = \ln x$
   - i. $x_1 = e,\ x_2 = e + 2$
   - ji. $x_1 = e,\ x_2 = e + .1$
   - ii. $x_1 = e,\ x_2 = e + .001$

c. $f(x) = e^{\sin x}$
   - i. $x_1 = 0,\ x_2 = -1$
   - ii. $x_1 = 0,\ x_2 = -.001$
   - iii. $x_1 = 0,\ x_2 = -.00001$

8. Use differentials to estimate the following quantities.

a. $\sqrt{229}$

b. $(250.1)^5$

c. $\sin(\pi - .04)$

d. $\log_{10}(9.9)$

9. Assume that the radius of a sphere is measured with a percentage error of at most 2%. With what percentage error will the following quantities be computed ?

a. diameter of the sphere
b. volume of the sphere
c. surface area of the sphere

10. Use differentials to show that $\sqrt{4+h} \approx 2 + (1/4)h$ for small h.

11. Use differentials to show that $\frac{1}{(8+h^3)^{1/3}} \approx 1/2 - (1/48)h^3$ for small h.

12. Use differentials to show that $\ln(1+h^2) \approx h^2$ for small h.

13. A basketball has diameter 10 inches. A coating of ice 1 inch thick covers the basketball.

a.) Use differentials to *estimate* the volume of ice around the basketball.
b.) Compute the *exact* volume of ice around the basketball
c.) What is the percentage error of the estimate relative to the exact value ?

14. Use basic properties of logarithms and exponents to prove that $\log_c x = \frac{\log_b x}{\log_b c}$, where b and c are any bases. Use this fact and your calculator to compute $\log_7 21$.