ESP
Kouba
Worksheet 17

1. Let $y = (\sin(x/2))^x + 5^x$. Compute y' at $x = \pi$.

2. Assume that y is a function of x and $y^3 + xy = 3y^2$. Compute y'' at the point $(0, 3)$.

3. Differentiate.

a. $y = \tan x + \arctan x$

b. $y = \sin\sqrt{x} - \arccos\sqrt{x}$

c. $y = \cot(\sin(5x)) + \operatorname{arcsec}(\csc x)$

d. $y = \ln(\arctan(\ln x))$

e. $y = \log_4(x \cdot 5^{3x})$

f. $y = \log_3(x^2 + e^{-x})$

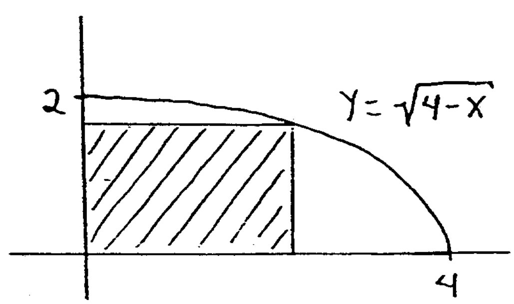
g. $y = \left(\dfrac{x+1}{3x-2}\right)^{5+x}$

h. $\log_x y = e^x$

i. $(xy)^{x^2} = (\tan y)^{xy^3}$

4. A rectangle is to be inscribed in the first quadrant below the graph of $y = \sqrt{4-x}$. Determine the dimensions of the rectangle of

a. maximum area.
b. maximum perimeter.
c. maximum sum of area and perimeter.

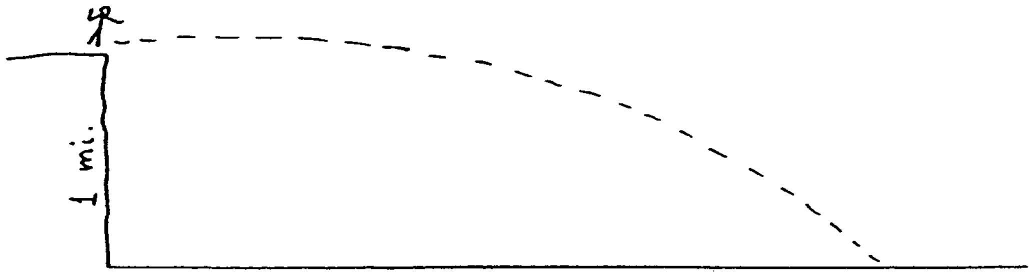


5. Evaluate the following limits.

a. $\displaystyle\lim_{n \to -\infty} \left(\frac{n+1}{n+2}\right)^{7n}$

b. $\displaystyle\lim_{n \to +\infty} \left(\frac{n^3}{1+n^3}\right)^{n}$

6. A baseball is fired horizontally from the top of a cliff, which is one mile high, at 100 miles per hour. See diagram.



a. How long does it take for the baseball to reach the ground ?
b. How far away from the base of the cliff does the baseball land ?
c. What is the " vertical velocity " of the baseball as it strikes the ground ?

7. Consider the function $f(x) = x^3 - 2x^2 + 3/2$.

a. Sketch the graph of f.
b. Use the Intermediate-Value Theorem to prove that $f(x) = 0$ has a solution r.

8. Prove that there is some number c , $3 < c < 4$, satisfying

$$\frac{4c^3}{c^4+1} = \ln(257/82)$$

HINT : Consider the function $f(x) = \ln(x^4 + 1)$.

9. For each of the following functions determine the x-values for which f is increasing, decreasing, concave up, and concave down. Indicate all maximum, minimum, and inflection points and intercepts. Neatly sketch the graph of f.

a. $y = xe^x$

b. $y = x \ln x$

c. $y = e^x + e^{-x}$

10. Use L'Hopital's rule to evaluate the following limits.

a. $\lim_{x \to 0} \frac{\sin x}{x}$

b. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

c. $\lim_{x \to 2} \frac{x^4 - 16}{\sqrt{x} - \sqrt{2}}$

d. $\lim_{x \to 0} \frac{\tan x}{x + \sin x}$

e. $\lim_{x \to 1} \frac{e^{x-1} - 2^{x-1}}{x^2 - x}$

f. $\lim_{x \to 0} \frac{x^2 \sin x + x \sin x}{x + 1 - \cos x}$

g. $\lim_{x \to 1} \frac{x \ln x + 1 - x}{(x-1)^2}$

h. $\lim_{x \to \frac{\pi}{2}} \frac{\tan x}{1 + \sec x}$

i. $\lim_{x \to +\infty} \frac{2^x + 2x}{5^x}$

j. $\lim_{x \to +\infty} \frac{x^3}{10^x}$

k. $\lim_{x \to 0} \frac{x e^x \cos^2 6x}{e^{2x} - 1}$

l. $\lim_{x \to +\infty} \frac{e^x - 1/x}{e^x + 1/x}$

m. $\lim_{x \to 0} \frac{\arcsin x}{\arctan 2x}$

n. $\lim_{x \to 0} \left\{ \frac{1}{1 - \cos x} - \frac{2}{x^2} \right\}$

o. $\lim_{x \to 0} \frac{\sin^2 x - x^2}{(e^{x^2} - 1)^2}$

p. $\lim_{x \to +\infty} \{\ln x\}^{1/x}$

q. $\lim_{x \to 0^+} \{\sin x\}^{1/x}$

r. $\lim_{x \to 0} (1 + x)^{1/x}$

s. $\lim_{n \to +\infty} (1 + 5/n)^{5n}$

t. $\lim_{n \to +\infty} (1 + n)^{1/n}$

u. $\lim_{x \to 0^+} x^2 \ln x$

v. $\lim_{x \to 0^+} \{\tan x\}^{\sqrt{x/3}}$

11. With each of the following functions are given numbers x_1 (the initial x-value) and x_2 (the final x-value). Compute the associated exact change in functional value, Δf, and the differential of f (approximate change in functional value), df.

a. $f(x) = x^3 + x - 1$
   i. $x_1 = 1,\ x_2 = 4$
   ii. $x_1 = 1,\ x_2 = 2$
   ii. $x_1 = 1,\ x_2 = 1.1$
   iv. $x_1 = 1,\ x_2 = 1.01$

b. $f(x) = \ln x$
   i. $x_1 = e,\ x_2 = e + 2$
   ii. $x_1 = e,\ x_2 = e + .1$
   ii. $x_1 = e,\ x_2 = e + .001$

c. $f(x) = e^{\sin x}$
   i. $x_1 = 0,\ x_2 = -1$
   ii. $x_1 = 0,\ x_2 = -.001$
   iii. $x_1 = 0,\ x_2 = -.00001$

12. Use differentials to estimate the following quantities.

a. $\sqrt{229}$

b. $(250.1)^5$

c. $\sin(\pi - .04)$

d. $\log_{10}(9.9)$

13. Assume that the radius of a sphere is measured with a percentage error of at most 2% . With what percentage error will the following quantities be computed ?

a. diameter of the sphere
b. volume of the sphere
c. surface area of the sphere