ESP
Kouba
Worksheet 3

1. Determine the following limits.

a. $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x - 1}$

b. $\lim_{x \to +\infty} \dfrac{x^2 - 3x}{2x^2 + x + 7}$

c. $\lim_{x \to 1^+} \dfrac{x}{x - 1}$

d. $\lim_{x \to -\infty} \dfrac{x^2 + 7x}{x + 13}$

e. $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$

f. $\lim_{x \to +\infty} \dfrac{x - 100{,}000}{x^5 + 1}$

g. $\lim_{x \to +\infty} (1 + \cos x)$

h. $\lim_{x \to -\infty} (\pi - \arctan x)$

i. $\lim_{x \to 0^+} \dfrac{x}{\sqrt{x^2 + x}}$

j. $\lim_{x \to 2} \dfrac{x^4 - 16}{x - 2}$

k. $\lim_{x \to -1} \dfrac{|x^2 - 1|}{x^2 - 1}$

l. $\lim_{x \to +\infty} (\sqrt{x^2 + 1} - x)$

2. Let $g(x) = \begin{cases} \cos 2x & \text{for } x \le 0 \\ \ln x & \text{for } 0 < x < 1 \\ \sqrt{x - 1} & \text{for } x \ge 1 \end{cases}$. Sketch the graph of g.

Determine the following limits.

a. $\lim_{x \to 0^-} g(x)$

b. $\lim_{x \to 0^+} g(x)$

c. $\lim_{x \to 1^-} g(x)$

d. $\lim_{x \to 1^+} g(x)$

e. $\lim_{x \to -\infty} g(x)$

f. $\lim_{x \to +\infty} g(x)$

3. Let $f(x) = x^2 + 4x + 3$ for $x \leq -2$. Find a function $g(x)$ so that $g(f(x)) = x$ for all admissable values of x.

4. What is the maximum number of parts into which a circle can be divided using 20 straight lines ?

5. Consider the following sequence :

$$1,\ 1+\frac{1}{1},\ 1+\frac{1}{1+\frac{1}{1}},\ 1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}},\ 1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}},\ \ldots$$

Determine the limit of this sequence.

6. Determine the following limits.

a. $\lim_{x \to -1} \dfrac{x^2 + 6x + 5}{x^3 + 1}$

b. $\lim_{x \to 5^+} \dfrac{5x}{\sqrt{x-5}}$

c. $\lim_{x \to 0^+} \dfrac{\sin x}{\sqrt{x}}$

d. $\lim_{x \to 4} \dfrac{x^{3/2} - 8}{x - 4}$

e. $\lim_{x \to +\infty} \dfrac{\sqrt{25x^2 + 3x + 5}}{4x}$

f. $\lim_{x \to -\infty} \dfrac{\sqrt{25x^2 + 3x + 5}}{4x}$

g. $\lim_{x \to 0} \dfrac{2x}{\sin x - x}$ (Use the fact that $\sin x / x \leq 1$ for $x \neq 0$.)

h. $\lim_{x \to \frac{\pi}{2}^+} \tan x$

i. $\lim_{x \to 0^+} \sin 1/x$

j. $\lim_{x \to 8} \dfrac{\sqrt[3]{x} - 2}{x - 8}$

7. Let $g(x) = \begin{cases} 1 + |x| & \text{for } x < 0 \\ x^3 & \text{for } 0 \le x \le 2 \\ 10 - x & \text{for } x > 2 \end{cases}$. Sketch its graph.

Determine the following limits.

a. $\lim_{x \to 0^+} g(x)$

b. $\lim_{x \to 0^-} g(x)$

c. $\lim_{x \to 2^+} g(x)$

d. $\lim_{x \to 2^-} g(x)$

e. $\lim_{x \to 0} g(x)$

f. $\lim_{x \to 2} g(x)$

g. $\lim_{x \to +\infty} g(x)$

h. $\lim_{x \to -\infty} g(x)$