ESP
Kouba
Worksheet 4

1. Determine the following limits.

   a. $\displaystyle\lim_{\theta \to 0}\ \frac{\sin 2\theta}{3\theta}$

   b. $\displaystyle\lim_{\theta \to 0}\ \frac{\tan 3\theta}{2\theta}$

   c. $\displaystyle\lim_{\theta \to 0}\ \frac{1-\cos\theta}{\sin\theta}$

   d. $\displaystyle\lim_{\theta \to 2}\ \frac{\sin(5\theta-10)}{3\theta-6}$

   e. $\displaystyle\lim_{\theta \to \frac{\pi}{4}}\ \frac{\tan\theta-1}{\theta-\pi/4}$

   f. $\displaystyle\lim_{\theta \to 0}\ \frac{\cos 3\theta-1}{\theta}$

   g. $\displaystyle\lim_{\theta \to 0}\ \frac{1-\cos^2\theta}{\theta}$

   h. $\displaystyle\lim_{h \to 0}\ \frac{\sin(\theta+h)-\sin\theta}{h}$

2. Sketch the graph of $g(x) = \begin{cases} \sin x & \text{for } x \le 0 \\ \ln\sqrt{x} & \text{for } 0 < x < e^2 \\ \dfrac{1}{1+x^2} & \text{for } x \ge e^2 \end{cases}$

3. Sketch the graph of $f(x) = \begin{cases} \dfrac{x^3+1}{x^2-1} & \text{for } x \ne 1, -1 \\ -3/2 & \text{for } x = -1 \\ 3 & \text{for } x = 1 \end{cases}$

4. Sketch the graph of a function which will satisfy all of the following conditions :

   a. $f(0) = 10$

   b. $f(2) = 0$

   c. $\displaystyle\lim_{x \to 2^-} f(x) = -\infty$

   d. $\displaystyle\lim_{x \to 3^-} f(x) = 1$

   e. $\displaystyle\lim_{x \to 5^+} f(x) = 3$

   f. $\displaystyle\lim_{x \to -2^+} f(x) = +\infty$

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5. For each part below, give an example of a function (give a specific formula) f satisfying :

a. $\lim\limits_{x \to 3^+} f(x) = +\infty = \lim\limits_{x \to 3^-} f(x)$

b. $\lim\limits_{x \to 3^+} f(x) = +\infty$ and $\lim\limits_{x \to 3^-} f(x) = -\infty$

6. Sketch a graph for each of the following. *Use intercepts and appropriate limits .*

a. $y = \dfrac{2x}{x-1}$

b. $y = \dfrac{x^2-x}{x^2+1}$

c. $y = \dfrac{x^2-1}{x+1}$

d. $y = \dfrac{|x^2-4|}{x^2-4}$

e. $y = \dfrac{x^2}{x^2-9}$

f. $y = \dfrac{x^3}{x^2-1}$

g. $y = \dfrac{x}{x^2-1}$

7. Without lifting your pencil, connect the sixteen dots using six straight lines.

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