ESP
Kouba
Worksheet 4 Solutions

See alternate rigorous solution at end of this solution set

1.) a.) $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta} = \lim_{\theta \to 0} \frac{2}{3} \cdot \frac{\sin(2\theta)}{(2\theta)} = \frac{2}{3}(1) = \frac{2}{3}$

b.) $\lim_{\theta \to 0} \frac{\tan 3\theta}{2\theta} = \lim_{\theta \to 0} \frac{3}{2} \cdot \frac{\sin 3\theta}{3\theta} \cdot \frac{1}{\cos 3\theta} = \frac{3}{2} \cdot 1 \cdot \frac{1}{1} = \frac{3}{2}$

c.) $\lim_{\theta \to 0} \frac{1-\cos\theta}{\sin\theta} = \lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{\theta}{\sin\theta}$

$= \lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{1}{\frac{\sin\theta}{\theta}} = 0 \cdot \frac{1}{1} = 0$

d.) $\lim_{\theta \to 2} \frac{\sin(5\theta - 10)}{3\theta - 6} = \lim_{\theta \to 2} \frac{5}{3} \cdot \frac{\sin 5(\theta - 2)}{5(\theta - 2)} = \frac{5}{3} \cdot 1 = \frac{5}{3}$

* e.) Use a calculator to get $\lim_{\theta \to \frac{\pi}{4}} \frac{\tan\theta - 1}{\theta - \frac{\pi}{4}} = 2$

f.) $\lim_{\theta \to 0} \frac{\cos 3\theta - 1}{\theta} = \lim_{\theta \to 0} 3 \cdot \frac{\cos(3\theta) - 1}{(3\theta)} = 3 \cdot 0 = 0$

g.) $\lim_{\theta \to 0} \frac{1-\cos^2\theta}{\theta} = \lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} \cdot (1+\cos\theta) = 0 \cdot (2) = 0$

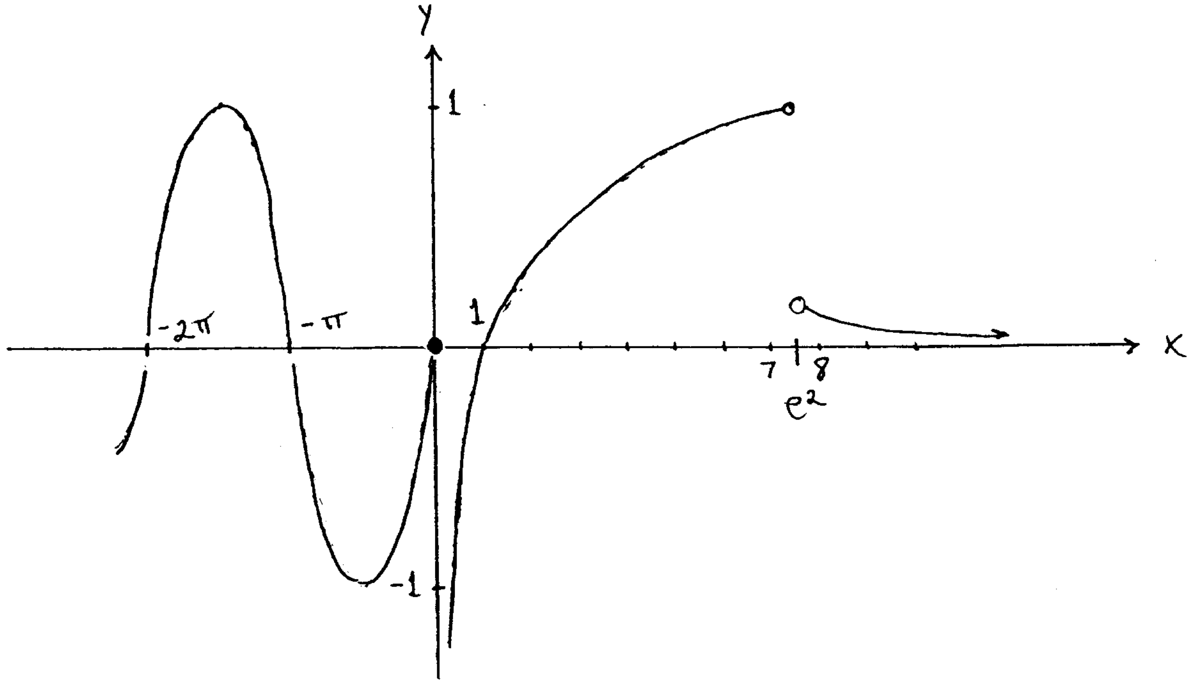
h.) $\lim_{h \to 0} \frac{\sin(\theta + h) - \sin\theta}{h} = \lim_{h \to 0} \frac{\sin\theta \cdot \cos h + \cos\theta \cdot \sin h - \sin\theta}{h}$

$= \lim_{h \to 0} \frac{\sin\theta(\cos h - 1) + \cos\theta \cdot \sin h}{h}$

$= \lim_{h \to 0} \left\{ \sin\theta \cdot \frac{\cos h - 1}{h} + \cos\theta \cdot \frac{\sin h}{h} \right\}$

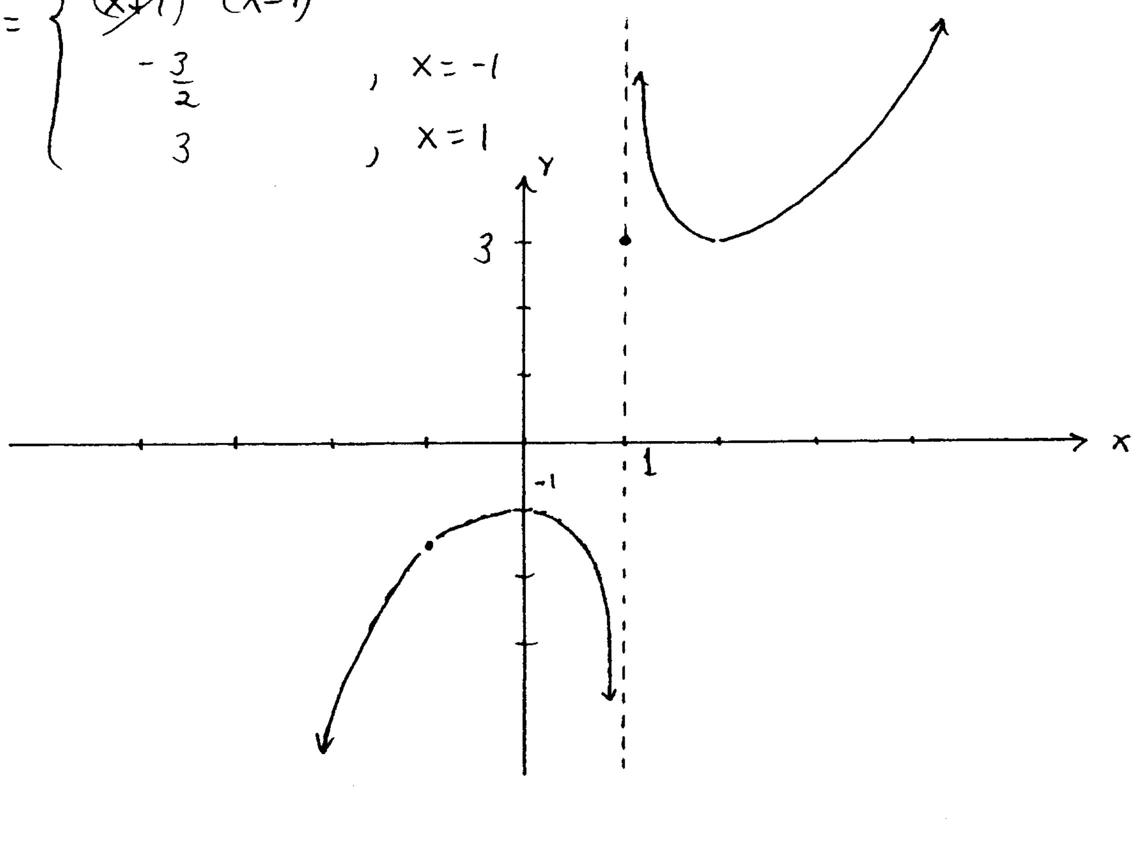
$= \sin\theta \cdot 0 + \cos\theta \cdot 1 = \cos\theta$

2.)

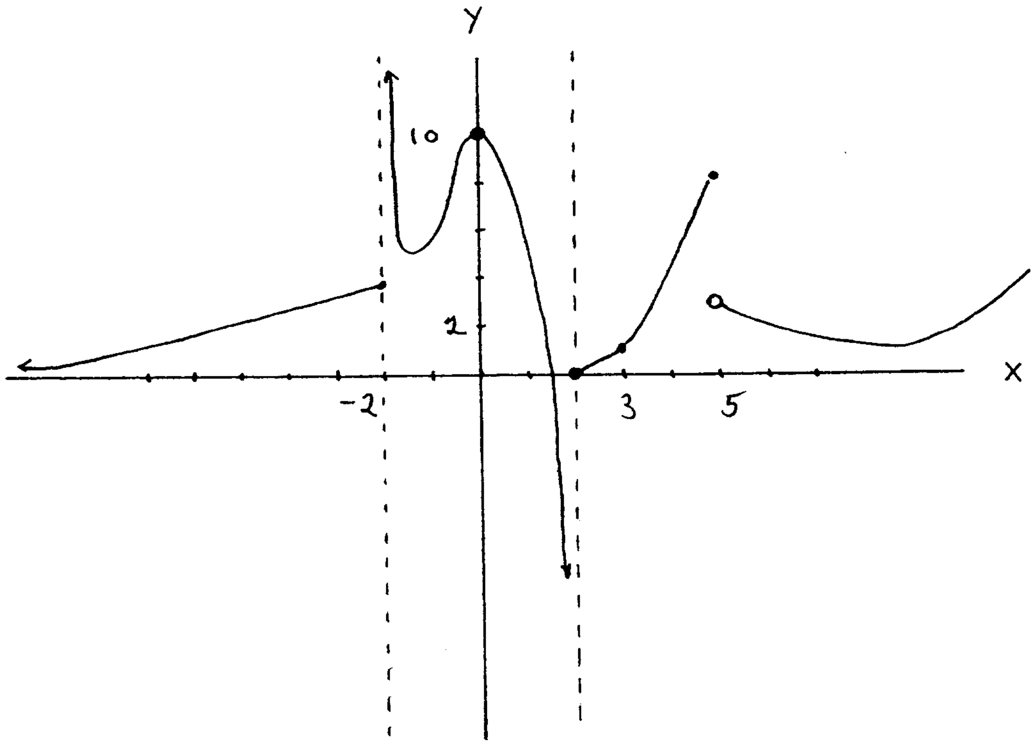


3.)

$$f(x) = \begin{cases} \dfrac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}\ (x-1)}, & x \neq 1, -1 \\ -\dfrac{3}{2}, & x = -1 \\ 3, & x = 1 \end{cases}$$

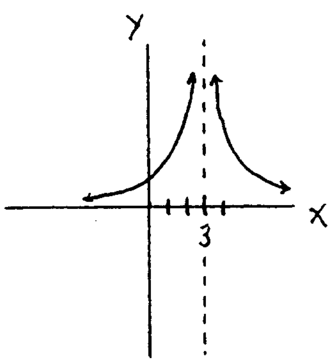


4.)



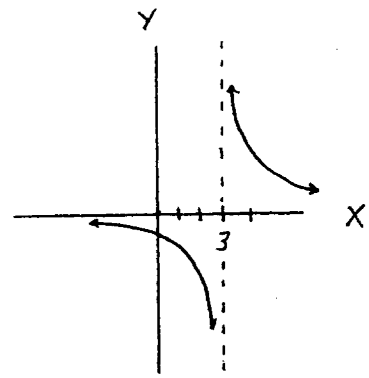
5.) a.)

$f(x) = \dfrac{1}{(x-3)^2}$

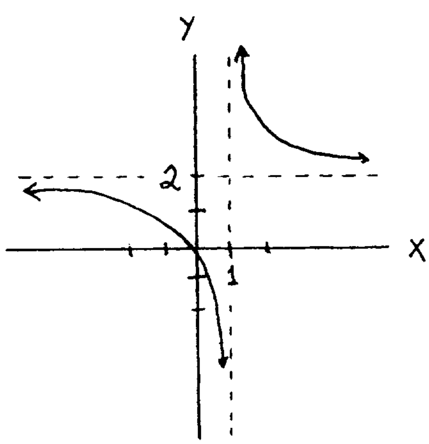


b.)

$f(x) = \dfrac{1}{x-3}$

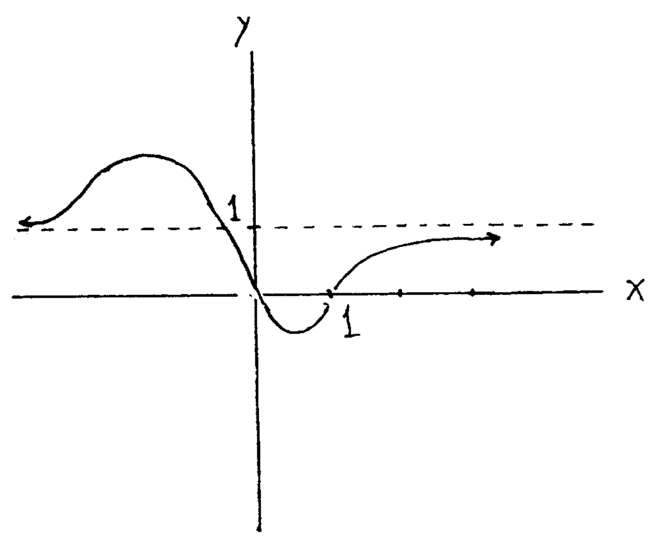


6.) a.)



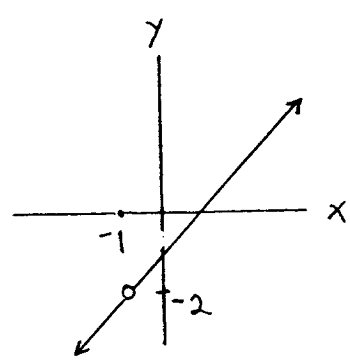
$Y = \dfrac{2x}{x-1}$

b.)



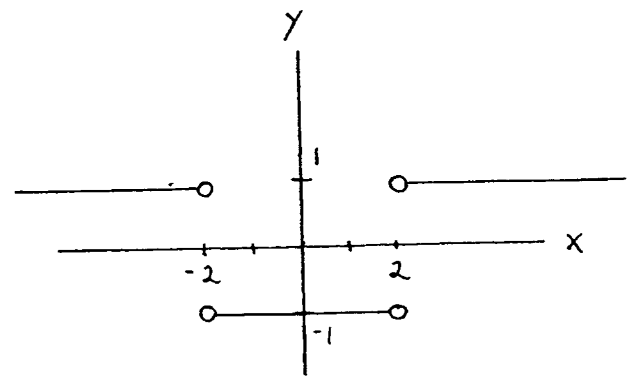
$Y = \dfrac{x \cdot (x-1)}{x^2+1}$

c.)



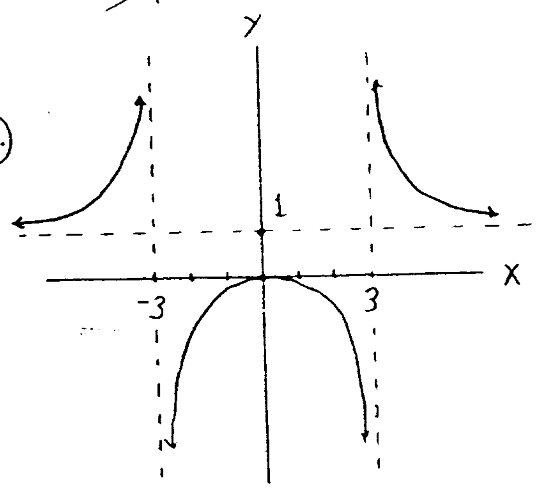
$$Y = \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = x-1 \quad \text{for } x \neq -1$$

d.)



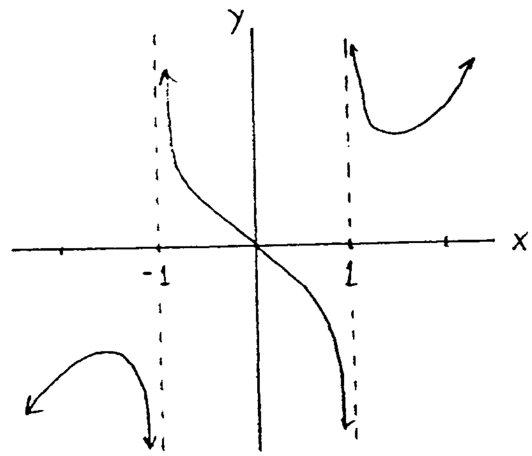
$$Y = \frac{|(x-2)(x+2)|}{(x-2)(x+2)}$$

e.)



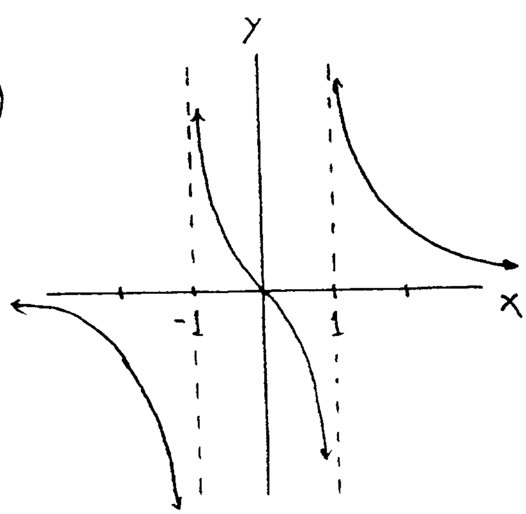
$$Y = \frac{x^2}{x^2-9} = \frac{x^2}{(x-3)(x+3)}$$

f.)



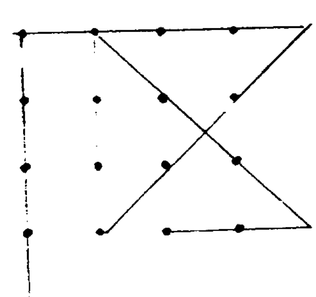
$$Y = \frac{x^3}{x^2-1} = \frac{x^3}{(x-1)(x+1)}$$

g.)



$$Y = \frac{x}{x^2-1} = \frac{x}{(x-1)(x+1)}$$

7.)



* Alternate solution to 1.) e.) $\lim_{\theta \to \frac{\pi}{4}} \dfrac{\tan\theta - 1}{\theta - \frac{\pi}{4}}$:

Let $u = \theta - \frac{\pi}{4}$ so that $\theta = u + \frac{\pi}{4}$ and

$$\lim_{\theta \to \frac{\pi}{4}} \frac{\tan\theta - 1}{\theta - \frac{\pi}{4}} = \lim_{u \to 0} \frac{\tan\left(u + \frac{\pi}{4}\right) - 1}{u}$$

$$= \lim_{u \to 0} \frac{\dfrac{\tan u + \tan\frac{\pi}{4}}{1 - \tan u \cdot \tan\frac{\pi}{4}} - 1}{u} = \lim_{u \to 0} \left(\frac{\tan u + 1}{1 - \tan u} - 1\right) \cdot \frac{1}{u}$$

$$= \lim_{u \to 0} \frac{2\tan u}{(1 - \tan u)\, u} = \lim_{u \to 0} 2 \cdot \left(\frac{\sin u}{u}\right) \cdot \frac{1}{\cos u} \cdot \frac{1}{1 - \tan u}$$

$$= 2 \cdot (1) \cdot 1 \cdot 1 = 2 .$$