ESP
Kouba
Worksheet 5

1. Determine the following limits.

    a. $\lim\limits_{x \to 0} \dfrac{|x|}{x}$

    b. $\lim\limits_{x \to 1} \dfrac{x^{16} - 1}{x - 1}$

    c. $\lim\limits_{x \to +\infty} \dfrac{1 + \sin x}{x^2}$

    d. $\lim\limits_{x \to 0} \dfrac{x}{1 - \sqrt{x^2 + 1}}$

2. Let $f(x) = \begin{cases} e^{-1/x^2} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$.

    a.   Determine the following limits.

        i. $\lim\limits_{x \to +\infty} f(x)$

        ii. $\lim\limits_{x \to -\infty} f(x)$

        iii. $\lim\limits_{x \to 0} f(x)$

    b.   Sketch the graph of f .

3. State and explain why and where (what x-values) the following functions are continuous.

    a. $f(x) = x^3 + \sin x$ b. $f(x) = 2x \cos x$

    c. $f(x) = \dfrac{3x^5}{x^2 - 4x}$ d. $y = \tan x$

    e. $y = \dfrac{3}{1 + \cos x}$ f. $g(x) = \sec 2x$

    g. $h(x) = (5 + \cos x)^5$ h. $h(x) = [(4x^3 - 1)^{20} + 2]^{10}$

    i. $f(x) = \begin{cases} x - 1 & \text{for } x \geq 2 \\ 2x^2 & \text{for } 0 < x < 2 \\ \dfrac{x}{x - 1} & \text{for } x < 0 \end{cases}$

4. Determine all values of c guaranteed by the Intermediate-Value Theorem for each of the following functions f , intervals [a, b] , and values m. Illustrate each with a graph.

    a. $f(x) = x^5 + 1$ on $[-1, 2]$, $m = 3$

    b. $f(x) = 3 - x^2$ on $[-2, 4]$, $m = -1$

    c. $f(x) = \sqrt{x - 2}$ on $[3, 11]$, $m = 2.3$

    d. $f(x) = \dfrac{x + 1}{x - 1}$ on $[1.1, 4]$, $m = 19$

    e. $f(x) = 2 \sin 3x$ on $[0, \frac{\pi}{6}]$, $m = 1$

    f. $f(x) = 1 - e^{-2x}$ on $[-\ln 10, 0]$, $m = -\pi$

    g. $f(x) = \dfrac{2x - 6}{x^2 + 1}$ on $[0, 1]$, $m = -2$

    h. $f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ 2 - x & \text{for } x > 1 \end{cases}$ on $[-1, 2]$, $m = 1/2$

2