ESP
Kouba
Worksheet 7

1. Determine the x-values for which the following functions are continuous.

a. $f(x) = \sin x$

b. $f(x) = \dfrac{1}{\sin x}$

c. $f(x) = \dfrac{x^4 - 1}{x^2 - 1}$

d. $f(x) = \begin{cases} \dfrac{x^4-1}{x^2-1} & \text{for } x \neq 1, -1 \\ 2 & \text{for } x = 1 \\ 3 & \text{for } x = -1 \end{cases}$

e. $f(x) = \begin{cases} x^2 + x & \text{for } x < 0 \\ \dfrac{\sin x}{\sqrt{x}} & \text{for } 0 \leq x < 2\pi \\ 0 & \text{for } x > 2\pi \end{cases}$

2. Assume that the total distance (miles) traveled by a bicyclist after time t (hours) is $T = 3t^2$.

a. Determine the average velocity of the bicyclist over the following intervals of time.

i. $t = 1$ to $t = 4$
ii. $t = 1$ to $t = 2$
iii. $t = 1$ to $t = 1.1$
iv. $t = 1$ to $t = 1.01$

b. Determine the exact velocity of the bicyclist when $t = 1$.

3. Position a wire, three centimeters long, on the positive x-axis with leftmost end at the origin. Assume that the left x centimeters of this wire have mass $\sqrt{x}$ grams and that the units for density shall be grams per centimeter.

a. Determine the average density of the wire on the following intervals.

i. $[2, 3]$
ii. $[2, 2.5]$
iii. $[2, 2.1]$
iv. $[2, 2.01]$

b. Determine the exact density of the wire at $x = 2$.

4. Consider the graph of $y = \dfrac{1}{x-1}$.

a. Determine the slope of the secant lines joining points determined by the following pairs of x-values.

i. $x = 3/2$, $x = 3$
ii. $x = 3/2$, $x = 2$
iii. $x = 3/2$, $x = 7/4$
iv. $x = 3/2$, $x = 25/16$

b. Determine the slope of the line tangent to the graph of $y = \dfrac{1}{x-1}$ at $x = 3/2$.

5. Use $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to compute the derivative of each of the following functions.

a. $f(x) = x - x^2$

b. $f(x) = \dfrac{x+1}{2x-3}$

c. $g(x) = \sqrt{x - 5}$

d. $h(x) = x - \frac{1}{x^2}$

e. $f(x) = \cos 4x$

f. $f(x) = \begin{cases} x^2 & \text{for } x \geq 0 \\ 1/2\ x & \text{for } x < 0 \end{cases}$

6. A spaceship is traveling along the curve $y = x^2$. At some point along this path, a payload will be released from the spaceship and travel through space along a line tangent to the curve. Where should the payload be released in order to be intercepted at a space station positioned at the point (3, 2) ?

7. Verify with a careful and complete explanation that the equation $1.9^x = x^2$ has at least 3 solutions.

8. a. Use the Intermediate-Value Theorem to show that $5x^3 - x + 7 = 0$ has at least one solution.

b. Let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial of odd degree n and with lead coefficient $a_n > 0$. Show that there is at least one real number r satisfying $P(r) = 0$.

9. Use the Intermediate-Value Theorem to show that $\frac{1}{x+3} - e^x = 0$ has a solution in the interval $[-1, 0]$.

10. Determine whether or not $x^3 = 2^x$ has a solution.

11. Let f and g be two continuous functions defined on the closed interval [a, b]. Assume that

$$f(a) < g(a) \quad \text{and} \quad f(b) > g(b).$$

Show that there is a number c in the open interval (a, b) satisfying

$$f(c) = g(c).$$

12. Assume that f and g are continuous functions and that $g(x) > 0$ on the interval [a, b]. In addition, assume that

$$f(a) = g(a) \text{ and } f(b) < 0.$$

Show that there is some number c, $a \le c \le b$, satisfying

$$2f(c) = g(c).$$

HINT: Consider the function $h(x) = f(x)/g(x)$.

13. How many numbers are in the following list and what is the sum of these numbers?

$$1, 5, 9, 13, 17, \cdots, 12045$$