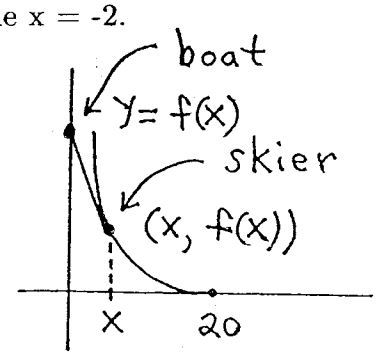
Math 21B Kouba
Discussion Sheet 8

1.) Use integration to derive the equation for the area of a circle of radius r.
2.) Use integration to derive the equation for the volume of a sphere of radius r.
3.) Find the area of the region bounded by the graphs of
   a.) $y = (\frac{-1}{2})x$, $y = x$, and $y = 2 - x$.
   b.) $y = e^{2x}$ and $y = \ln x$ on the interval [1, 3].
   c.) $y = 2^x$, $y = 2^{-x}$, and $x = 1$.
   d.) $x = (y + 2)\, y^2$, $y = -2$, and $x = 3$.
   e.) $y = \sqrt{x}$, $y = 1/x$, and $y = (3/4)x - 1$.
4.) Find the area of the region inside the circle $x^2 + y^2 = 25$ and above the line $y = (-1/7)x + 25/7$.
5.) The flat base of a solid lies in the region bounded by the graphs of $y = x^2, y = 0$, and $x = 2$. Set up, but do not evaluate an integral which represents the volume of the solid if cross-sections taken perpendicular to the x-axis are
   a.) squares. b.) semi-circles.
6.) Consider a hemispherical solid with a flat circular base of radius 1 foot. Set up but do not evaluate an integral which represents the total weight of the solid if density is given by $\sqrt{x^3 + 4}$ lbs./ft.^{3}, where
   a.) x is the distance from the center of the hemisphere. (Assume that the surface area of a hemisphere of radius r is $2\pi r^2$.)
   b.) x is the distance from the flat base.
7.) The flat base of a solid lies in the region bounded by the graphs of $y = \sqrt{x}$, $y = 0$ and $x = 4$. Set up but do not evaluate an integral which represents the volume of the solid if cross-sections taken perpendicular to the x-axis are
   a.) equilateral triangles. b.) right triangles (leg down) of perimeter 6.
8.) Consider the region bounded by the graphs of $y = \ln x$, $y = 0$, and $x = e$. Compute the area of the region using
   a.) vertical cross-sections. b.) horizontal cross-sections.
9.) Consider the region given in problem 5.). Use the DISC METHOD to set up (but not evaluate) integrals which represent the volume of the solid formed by revolving the region about the a.) x-axis. b.) line y = 5. c.) y-axis d.) line x = -2.
10.) A motor boat is resting at the origin, (0, 0), and a skier, tethered to the boat with a 20-ft. rope, is resting at the point (20, 0). The boat then begins moving along the y-axis, pulling the skier along the unknown path, $y = f(x)$. Use integration to find an equation for this path.



11.) A flat equilateral triangular plate of uniform density has a total mass of 25 kg. and edge length 2 m. Find the plate's kinetic energy if it rotates 8 radians per second about one of its edges.

" What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises."– G. Lichtenberg