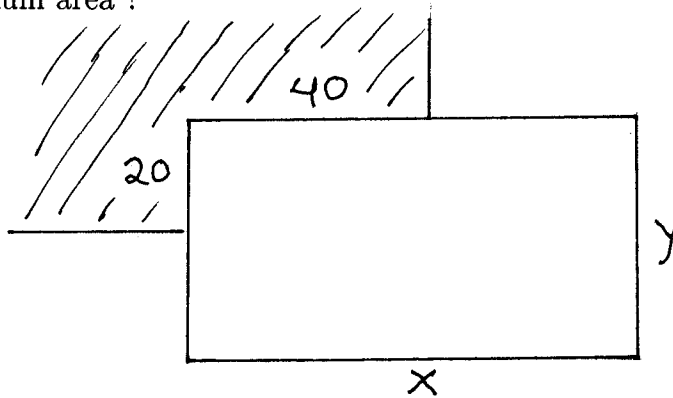Math 21A
Kouba
Challenge Discussion Sheet 7

1.) One hundred and forty (140) feet of fencing will be extended from a 20 ft. by 40 ft. corner of a building to create a rectangular pen. What dimensions x and y will result in the pen of maximum area ?

2.) Find that number so that 4 times itself exceeds its square by the largest amount.

3.) A rectangle is inscribed beneath the graph of $y = 12 - x^2$ and above the x-axis. Find the dimensions of the rectangle of
    a.) maximum area.   b.) maximum perimeter.
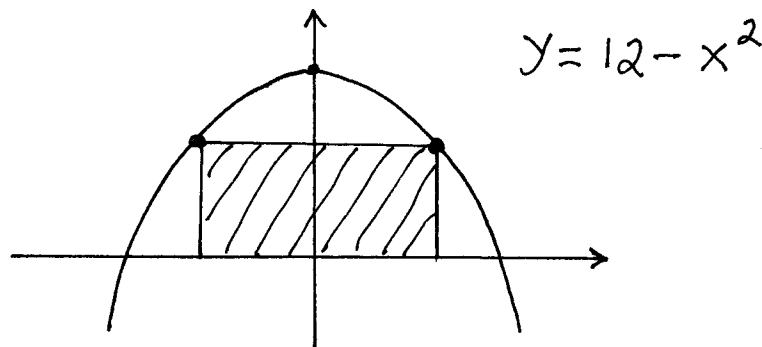
4.) A piece of wire 16 inches long is to be cut into two pieces. One piece is bent into a circle and the other is bent into a square. Where should the cut be made in order that the sum of the areas is a
    a.) minimum ?   b.) maximum ?

5.) Determine $y' = dy/dx$ for each.

    a.) $x^2 + y^2 = y + 3x$
    b.) $4 + \tan(x - y) = \sec(y^3)$
    c.) $\dfrac{x^2 y}{x + y^3} = \dfrac{y - 1}{x + 2}$

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6.) Compute the slope and concavity of the graph of $xy + y^3 = 8$ at $x = 0$. Sketch the graph near $x = 0$.

7.) Show that the hyperbolas $xy = 1$ and $x^2 - y^2 = 1$ intersect at right angles.

8.) The graph of the equation $(x^2 + y^2 - 4x)^2 = 4(x^2 + y^2)$ is given. It is called a Limacon of Pascal. Determine the slope of the line tangent to the graph at point A $(x = 1)$. CHALLENGE: Determine the slope(s) of the line(s) tangent to the graph at $(0,0)$.