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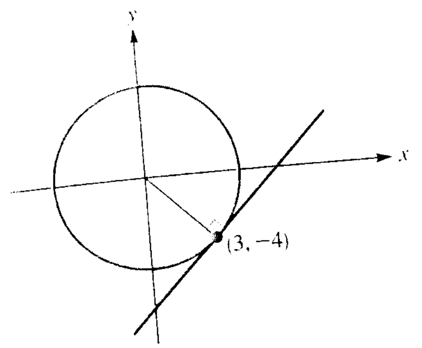


(a) Find an equation of the tangent line. *Hint:* Make use of the theorem from elementary geometry stating that the tangent line is perpendicular to the radius drawn to the point of contact.

(b) Find the intercepts of the tangent line.

(c) Find the length of the portion of the tangent line in quadrant IV.

For Exercises 51 and 52, you'll need to recall the following definitions and results from elementary geometry. In a triangle, a line segment drawn from a vertex to the midpoint of the opposite side is called a ***median.*** *The three medians of a triangle are* ***concurrent;*** *that is, they intersect in a single point. This point of intersection is called the* ***centroid*** *of the triangle. A line segment drawn from a vertex perpendicular to the opposite side is an* ***altitude.*** *The three altitudes of a triangle are concurrent; the point where the altitudes intersect is the* ***orthocenter*** *of the triangle.*

Ⓖ **51.** This exercise provides an example of the fact that the medians of a triangle are concurrent.

(a) The vertices of $\triangle ABC$ are as follows:

$$A(-4, 0) \quad B(2, 0) \quad C(0, 6)$$

Use a graphing utility to draw $\triangle ABC$. (Since $\overline{AB}$ coincides with the x-axis, you won't need to draw a line segment for this side.) *Note:* If the graphing utility you use does not have a provision for drawing line segments, you will need to determine an equation for the line in each case and then graph the line.

(b) Find the coordinates of the midpoint of each side of the triangle, then include the three medians in your picture from part (a). Note that the three medians do appear to intersect in a single point. Use the graphing utility to estimate the coordinates of the centroid.

(c) Using paper and pencil, find the equation of the medians from A to $\overline{BC}$ and from B to $\overline{AC}$. Then (using simultaneous equations from intermediate algebra), determine the exact coordinates of the centroid. How do these numbers compare with your estimates in part (b)?

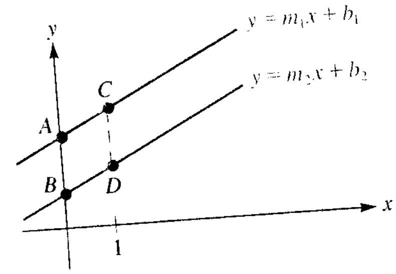
Ⓖ **52.** This exercise illustrates the fact that the altitudes of a triangle are concurrent. Again, we'll be using $\triangle ABC$ with vertices $A(-4, 0)$, $B(2, 0)$, and $C(0, 6)$. Note that one of the altitudes of this triangle is just the portion of the y-axis extending from $y = 0$ to $y = 6$; thus, you won't need to graph this altitude: it will already be in the picture.

(a) Using paper and pencil, find the equations for the three altitudes. (Actually, you are finding equations for the lines that coincide with the altitude segments.)

(b) Use a graphing utility to draw $\triangle ABC$ along with the three altitude lines that you determined in part (a). Note that the altitudes appear to intersect in a single point. Use the graphing utility to estimate the coordinates of this point.

(c) Using simultaneous equations (from intermediate algebra), find the exact coordinates of the orthocenter. Are your estimates in part (b) close to these values?

53. This exercise outlines a proof of the fact that two nonvertical lines are parallel if and only if their slopes are equal. The proof relies on the following observation for the given figure: The lines $y = m_1x + b_1$ and $y = m_2x + b_2$ will be parallel if and only if the two vertical distances AB and CD are equal. (In the figure, the points C and D both have x-coordinate 1.)



(a) Verify that the coordinates of A, B, C, and D are

$$A(0, b_1) \quad B(0, b_2) \quad C(1, m_1 + b_1) \quad D(1, m_2 + b_2)$$

(b) Using the coordinates in part (a), check that

$$AB = b_1 - b_2 \quad \text{and} \quad CD = (m_1 + b_1) - (m_2 + b_2)$$

(c) Use part (b) to show that the equation $AB = CD$ is equivalent to $m_1 = m_2$.

54. This exercise outlines a proof of the fact that two nonvertical lines with slopes m_1 and m_2 are perpendicular if and only if $m_1m_2 = -1$. In the following figure, we've assumed that our two nonvertical lines $y = m_1x$ and $y = m_2x$ intersect at the origin. [If they did not intersect there, we could just as well work with lines parallel to these that do intersect at (0, 0), recalling that parallel lines have the same slope.] The proof relies on the following geometric fact:

$$\overline{OA} \perp \overline{OB} \quad \text{if and only if} \quad (OA)^2 + (OB)^2 = (AB)^2$$

(a) Verify that the coordinates of A and B are $A(1, m_1)$ and $B(1, m_2)$.

(b) Show that

$$OA^2 = 1 + m_1^2$$
$$OB^2 = 1 + m_2^2$$
$$AB^2 = m_1^2 - 2m_1m_2 + m_2^2$$

(c) Use part

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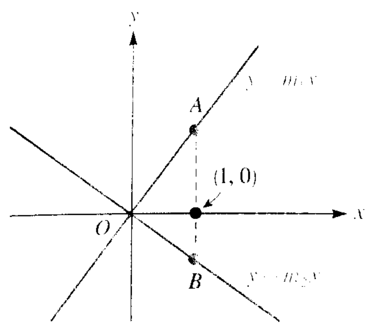
(c) Use part (b) to show that the equation

$$OA^2 + OB^2 = AB^2$$

is equivalent to $m_1 m_2 = -1$.

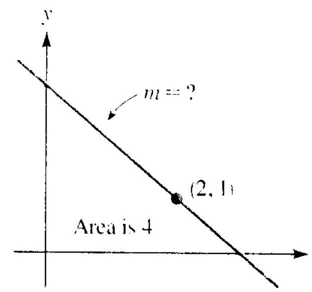


55. Verify the identity

$$(y_2 - y_1)/(x_2 - x_1) = (y_1 - y_2)/(x_1 - x_2)$$

What does this identity tell you about calculating slope?

56. Find the slope m of the line in the following figure.



MINI PROJECT THINKING ABOUT SLOPE

Working within a group or with the class at large, carry out the two activities indicated below. Then, on your own, write a summary of what you have learned.

1. What if the definition of slope that we gave in this section had instead been $m = \dfrac{y_2 + y_1}{x_2 + x_1}$? Try reworking Example 1 using this version of slope and describe what happens. Why is this unsatisfactory?
2. Why, where, and when did the use of the letter m for slope originate? See what you can find out by reading comments from mathematicians and math historians at the following Internet website:

   http://mathforum.org/epigone/math-history-list

   On the right-hand side of that web page, click on "search this discussion," and type in the word *slope*. As with many historical issues in many fields, don't expect one simple definitive answer. Summarize what you find out.

1.7 SYMMETRY AND GRAPHS. CIRCLES

Symmetry is a working concept. If all the object is symmetrical, then the parts must be halves (or some other rational fraction) and the amount of information necessary to describe the object is halved (etc.). —Alan L. Macay, Department of Crystallography, University of London

As you will see throughout this text, there are some basic techniques that help us to understand the essential features of a graph. In the box that follows, we introduce three types of *symmetry* that are useful in analyzing graphs.

1.7

between 1 and 2, closer to 2 than to 1; the larger x-intercept is between 4 and 5, closer to 4 than to 5. Figure 14(b) shows a close-up view of the smaller x-intercept. Evidently, it is closer to 1.680 than to 1.670, and so our estimate to two decimal places is 1.68. Exercise 62 asks you to use a graphing utility in a similar manner to show that the larger x-intercept is approximately 4.32.

(c) The easiest way to find exact expressions for the x-intercepts is to use equation (7). Setting $y = 0$ in equation (7) yields

$$(x-3)^2 + \frac{9}{4} = 4$$

$$(x-3)^2 = \frac{7}{4} \qquad \text{Check the arithmetic.}$$

$$x - 3 = \pm\sqrt{\frac{7}{4}} = \pm\frac{\sqrt{7}}{2}$$

$$x = 3 \pm \frac{\sqrt{7}}{2}$$

In summary, the two x-intercepts are $3 - \sqrt{7}/2$ and $3 + \sqrt{7}/2$. As you can check for yourself with a calculator, these last two expressions are approximately 1.68 and 4.32, respectively. These values are consistent with the numbers that we obtained graphically in part (b).

EXERCISE SET 1.7

A

In Exercises 1–6, the endpoints of a line segment $\overline{AB}$ are given. Sketch the reflection of $\overline{AB}$ about **(a)** *the x-axis;* **(b)** *the y-axis; and* **(c)** *the origin.*

1. $A(1, 4)$ and $B(3, 1)$
2. $A(-1, -2)$ and $B(-5, -2)$
3. $A(-2, -3)$ and $B(2, -1)$
4. $A(-3, -3)$ and $B(-3, -1)$
5. $A(0, 1)$ and $B(3, 1)$
6. $A(-2, -2)$ and $B(0, 0)$

In Exercises 7–24, graph the equation after determining the x- and y-intercepts and whether the graph possesses any of the three types of symmetry described on page 58.

7. $y = 4 - x^2$
8. $y = -x^3$
9. $y = -1/x$
10. $x = y^2 - 1$
11. $y = -x^2$
12. $y = 1/x^2$
13. $y = -1/x^3$
14. $y = |x| - 2$
15. $y = \sqrt{x^2}$
16. $y = x + 1$
17. $y = x^2 - 2x + 1$
18. $x = y^3 - 1$
19. $y^2 = 2x - 4$
20. $|y| = 2x - 4$
21. $y = 2x^2 + x - 4$
22. $y = 2x^2$
23. (a) $x + y = 2$ **(b)** $|x| + y = 2$
24. (a) $x + |y| = 2$ **(b)** $|x + y| = 2$

Ⓖ *In Exercises 25–38:*

(a) *Use a graphing utility to graph each equation in the standard viewing rectangle.*
(b) *Does the graph in part (a) appear to possess any of the three types of symmetry defined on page 58?*
(c) *In cases in which your answer to part (b) is Yes, adjust your viewing rectangle for a second, more careful inspection. (In Exercises 25–31, suggestions for the second look are provided.)*
(d) *In cases in which, after the second look, it still appears that the graph possesses symmetry, use an appropriate symmetry test from page 59 to settle the matter.*

25. $y = x^2 - 3x$ (second view: x from -2 to 5; y from -4 to 10)
26. $y = x^3 - 3x$ (second view: x from -3 to 3; y unchanged)
27. $y = 2^x$ (second view: x from -3 to 3; y from 0 to 8)
28. $y = 2^{|x|}$ (second view: x from -3 to 3; y from 0 to 8)
29. $y = 1/(x^2 - x)$ (second view: x from -2 to 2; y unchanged)
30. $y = 1/(x^3 - x)$ (second view: x from -2 to 2; y unchanged)
31. $y = x^2 - 0.2x - 15$ *Hint:* Look closely at the x-intercepts.
32. $y = x^2 - 2x^3$
33. $y = \sqrt{|x|}$
34. $y = \sqrt{|x|^3}$
35. $y = 2x - x^3 - x^5 + x^7$
36. $y = |2x - x^3 - x^5 + x^7|$
37. $y = x^4 - 10x^2 + \frac{1}{4}x$ *Suggestion:* For the second view, try zooming out in the y-direction.
38. $y = x^4 - 10x^2 + \frac{1}{4}$

In Exercises 39–42, specify the center and radius of each circle. Also, determine whether the given point lies on the circle.

39. $(x-1)^2 + (y-5)^2 = 169$; $(6, -7)$
40. $(x+4)^2 + (y+2)^2 = 20$; $(0, 1)$
41. $(x+8)^2 + (y-5)^2 = 13$; $(-5, 2)$
42. $x^2 + y^2 = 1$; $(1/2, \sqrt{3}/2)$

In Exercises 43–48, determine the center and the radius for the circle. Also, find the y-coordinates of the points (if any) where the circle intersects the y-axis.

43. $x^2 + y^2 = \sqrt{2}$
44. $x^2 + y^2 - 10x + 2y + 17 = 0$
45. $x^2 + y^2 + 8x - 6y = -24$
46. $4x^2 - 4x + 4y^2 - 63 = 0$
47. $9x^2 + 54x + 9y^2 - 6y + 64 = 0$
48. $3x^2 + 3y^2 + 5x - 4y = 1$

In Exercises 49 and 50, use the techniques shown in Example 7 to carry out the following procedures.

(a) *Find the center and radius of the circle.*
Ⓖ **(b)** *Use a graphing utility to graph the circle and to estimate (to two decimal places) the x-intercepts.*
(c) *Use algebra to find exact values for the x-intercepts, and then use a calculator to check that the results are consistent with the estimates in part (b).*

49. $16x^2 - 64x + 16y^2 + 48y - 69 = 0$
50. $3x^2 + x + 3y^2 + 3y - 1 = 0$

Ⓖ **51.** In the text we said that a line is an *asymptote* for a curve if the distance between the line and the curve approaches zero as we move farther and farther out along the line. In terms of graphing, this means that as we zoom out, the curve and the line eventually appear indistinguishable. In this exercise, we'll demonstrate this using the curve $y = -4/x$ (which we graphed in Figure 9). As indicated in the text, both the x- and y-axes are asymptotes for this curve. First, graph $y = -4/x$ using a viewing rectangle that extends from -5 to 5 in both the x- and the y-directions. Then take a second look using a viewing rectangle that extends from -30 to 30 in both the x- and y-directions. At this scale, you'll see that the curve is virtually indistinguishable from an asymptote when either $|x| > 8$ or $|y| > 8$.

Ⓖ **52.** **(a)** Graph the equation $y = 20/x$ using a standard viewing rectangle.
(b) Although both the x- and the y-axes are asymptotes for this curve, the graph in part (a) does not show this clearly. Take a second look, using a viewing rectangle that extends from -100 to 100 in both the x- and the y-directions. Note that the curve indeed appears indistinguishable from an asymptote when either $|x|$ or $|y|$ is sufficiently large.

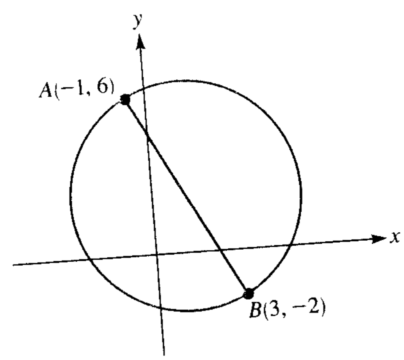
53. The center of a circle is the point $(3, 2)$. If the point $(-2, -10)$ lies on this circle, find the standard equation for the circle.

54. Find the standard equation of the circle tangent to the x-axis and with center $(3, 5)$. *Hint:* First draw a sketch.

55. Find the standard equation of the circle tangent to the y-axis and with center $(3, 5)$.

56. Find the standard equation of the circle passing through the origin and with center $(3, 5)$.

57. The points $A(-1, 6)$ and $B(3, -2)$ are the endpoints of a diameter of a circle, as indicated in the accompanying figure. Find the y-intercepts of the circle. *Hint:* Could you do the problem if you had the equation of the circle?

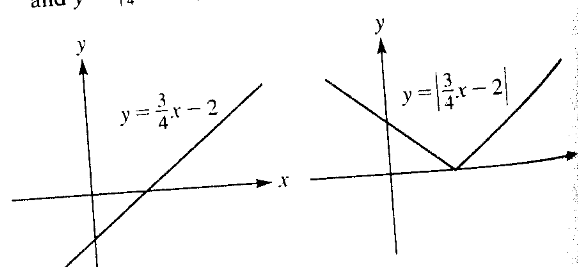


58. **(a)** Verify that the point $(3, 7)$ is on the circle

$$x^2 + y^2 - 2x - 6y - 10 = 0$$

(b) Find the equation of the line tangent to this circle at the point $(3, 7)$. *Hint:* A result from elementary geometry says that the tangent to a circle is perpendicular to the radius drawn to the point of contact.

59. The accompanying figure shows the graphs of $y = \frac{3}{4}x - 2$ and $y = |\frac{3}{4}x - 2|$.



(a) Determine the x- and y-intercepts for each graph.
(b) Which portions of the two graphs are identical? (Give your answer in terms of an interval along the x-axis.)
(c) Explain how the graph of $y = |\frac{3}{4}x - 2|$ can be obtained from that of $y = \frac{3}{4}x - 2$ by means of reflection

60. The accon
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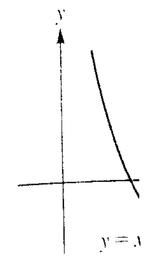

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