2.4

B

77. Suppose that after studying a corporation's records, a business analyst predicts that the corporation's monthly revenues R for the near future can be closely approximated by the equation

$$R = -0.0217x^5 + 0.626x^4 - 6.071x^3 + 25.216x^2 - 57.703x + 159.955 \qquad (1 \le x \le 12)$$

where R is the revenue (in thousands of dollars) for the month x, with $x = 1$ denoting January, $x = 2$ denoting February, and so on.

(a) According to this model, for which months will the monthly revenue be no more than $80,000?
Hint: You need to solve the inequality $R \le 80$. Round each key number to the nearest integer. (Why?)

(b) For which months, if any, will the monthly revenue be at least $120,000?

78. (Continuation of Exercise 77)

(a) Solve the inequality $R > 165$ to determine the months, if any, that the revenue will exceed $165,000.

(b) Are there any months when the revenue will fall below $45,000?

79. For which values of b will the equation $x^2 + bx + 1 = 0$ have real solutions?

80. The sum of the first n natural numbers is given by

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

For which values of n will the sum be less than 1225?

81. For which values of a is $x = 1$ a solution of the following inequality?

$$\frac{2a + x}{x - 2a} < 1$$

82. Solve $\dfrac{ax + b}{\sqrt{x}} > 2\sqrt{ab}$, where a and b are positive constants.

83. The two shorter sides in a right triangle have lengths x and $1 - x$, where $x > 0$. For which values of x will the hypotenuse be less than $\sqrt{17}/5$?

84. A piece of wire 12 cm long is cut into two pieces. Denote the lengths of the two pieces by x and $12 - x$. Both pieces are then bent into squares. For which values of x will the combined areas of the squares exceed 5 cm^2?

C

85. Find a nonzero value for c so that the solution set for the inequality

$$x^2 + 2cx - 6c < 0$$

is the open interval $(-3c, c)$.

86. Solve $(x - a)^2 - (x - b)^2 > (a - b)^2/4$, where a and b are constants and $a > b$.

PROJECT 1 WIND POWER

The relatively recent [in terms of human history] *transition to coal that began in Europe in the seventeenth century marked a major shift to dependence on a finite stock of fossilized fuels whose remaining energy is now equivalent to less than 11 days of sunshine. From a millennial perspective, today's hydrocarbon-based civilization is but a brief interlude in human history.*—Christopher Flavin and Seth Dunn, "Reinventing the Energy System" in *Vital Signs, 1999,* Lester R. Brown et al. (New York: W. W. Norton & Co., 1999)

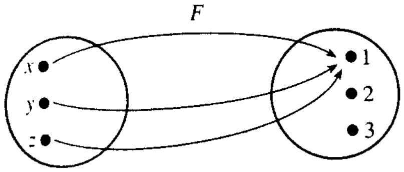
As the world's nonrenewable energy resources such as oil and coal start to become more scarce, alternative sources such wind and solar become increasingly important. In this project you'll calculate some estimates concerning wind power, an industry and technology that has grown rapidly over the past two decades. (Later, in Chapter 5, after studying exponential and logarithmic functions, we'll be able to make some rough estimates for possible depletion dates regarding resources such as oil and coal.) In working this project, you'll be reviewing math from the following sections of this book:

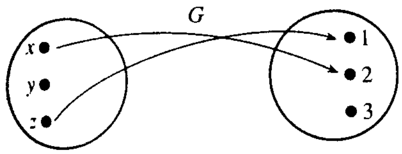
- 1.3 Solving Equations
- 1.6 Equations of Lines
- 2.3 Inequalities
- 2.4 More on Inequalities

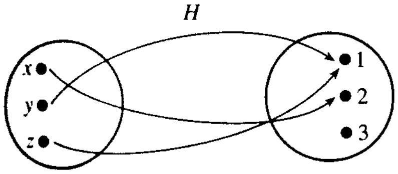
(*continues*)

3.1

6.





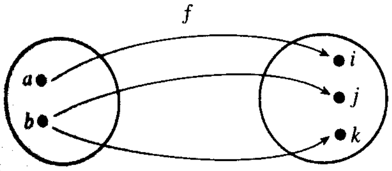


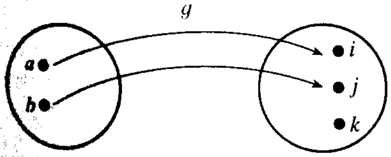
For Exercises 7 and 8, two sets D and C are defined as follows: $D = \{a, b\}$; $C = \{i, j, k\}$.

(a) *Which of the rules displayed in the figures represent functions from D to C?*

(b) *For each rule that represents a function, specify the range.*

7.





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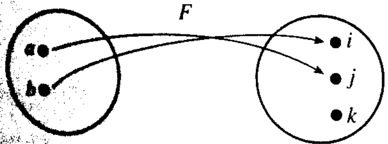
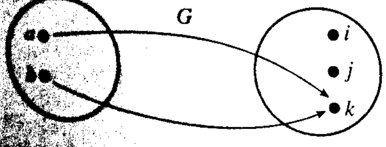




In Exercises 9–16, determine the domain of each function.

9. **(a)** $y = -5x + 1$
(b) $y = 1/(-5x + 1)$
(c) $y = \sqrt{-5x + 1}$
(d) $y = \sqrt[3]{-5x + 1}$

10. **(a)** $s = 3t + 12$
(b) $s = 1/(3t + 12)$
(c) $s = \sqrt{3t + 12}$
(d) $s = \sqrt[3]{3t + 12}$

11. **(a)** $f(x) = x^2 - 9$
(b) $g(x) = 1/(x^2 - 9)$
(c) $h(x) = \sqrt{x^2 - 9}$
(d) $k(x) = \sqrt[3]{x^2 - 9}$

12. **(a)** $F(t) = t^2 + 4t$
(b) $G(t) = 1/(t^2 + 4t)$
(c) $H(t) = \sqrt{t^2 + 4t}$
(d) $K(t) = \sqrt[3]{t^2 + 4t}$

13. **(a)** $f(t) = t^2 - 8t + 15$
(b) $g(t) = 1/(t^2 - 8t + 15)$
(c) $h(t) = \sqrt{t^2 - 8t + 15}$
(d) $k(t) = \sqrt[3]{t^2 - 8t + 15}$

14. **(a)** $F(x) = 2x^2 + x - 6$
(b) $G(x) = 1/(2x^2 + x - 6)$
(c) $H(x) = \sqrt{2x^2 + x - 6}$
(d) $K(x) = \sqrt[3]{2x^2 + x - 6}$

15. **(a)** $f(x) = (x - 2)/(2x + 6)$
(b) $g(x) = \sqrt{(x - 2)/(2x + 6)}$
(c) $h(x) = \sqrt[3]{(x - 2)/(2x + 6)}$

16. **(a)** $F(t) = (3t - 4)/(7 - 2t)$
(b) $G(t) = \sqrt{(3t - 4)/(7 - 2t)}$
(c) $H(t) = \sqrt[3]{(3t - 4)/(7 - 2t)}$

In Exercises 17–26, determine the domain and the range of each function.

17. $y = 4x - 5$

18. $y = 125 - 12x$

19. $y = 4x^3 - 5$

20. $y = 125 - 12x^3$

21. $g(x) = \dfrac{4x - 20}{3x - 18}$

22. $f(x) = \dfrac{1 - x}{x}$

23. **(a)** $f(x) = \dfrac{x + 3}{x - 5}$
(b) $F(x) = \dfrac{x^3 + 3}{x^3 - 5}$

24. **(a)** $g(x) = \dfrac{2x - 7}{3x + 24}$
(b) $G(x) = \dfrac{2x^3 - 7}{3x^3 + 24}$

25. $s = t^2 + 4$

26. $s = 2t^2 - 10$

27. Each of the following rules defines a function with domain the set of all real numbers. Express each rule in the form of an equation.

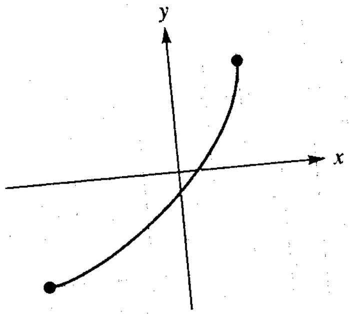
EXAMPLE
The rule *For each real number, compute its square* can be written $y = x^2$.

(a) For each real number, subtract 3 and then square the result.
(b) For each real number, compute its square and then subtract 3 from the result.
(c) For each real number, multiply it by 3 and then square the result.
(d) For each real number, compute its square and then multiply the result by 3.

28. Each of the following rules defines a function with domain equal to the set of all real numbers. Express each rule in words.

(a) $y = 2x^3 + 1$
(b) $y = 2(x + 1)^3$
(c) $y = (2x + 1)^3$
(d) $y = (2x)^3 + 1$

7.



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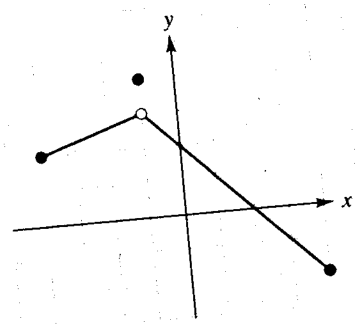

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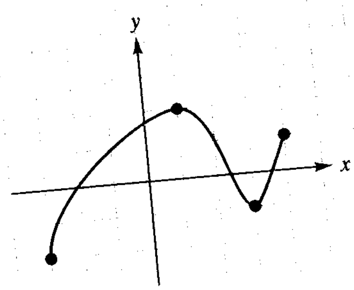

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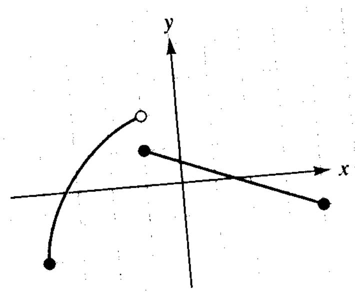

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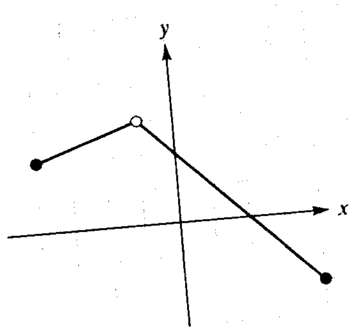

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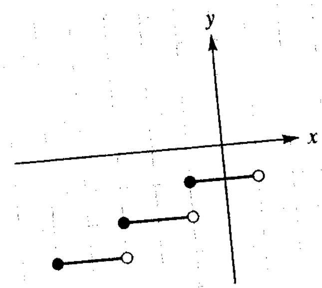

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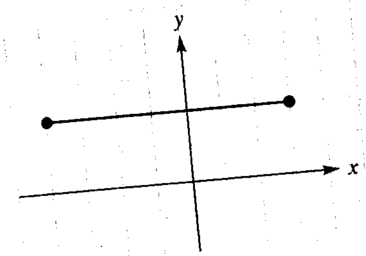

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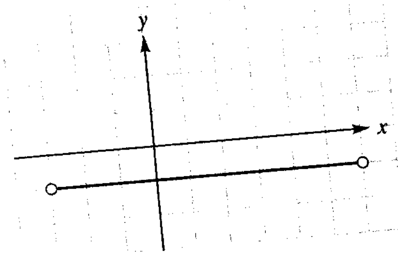
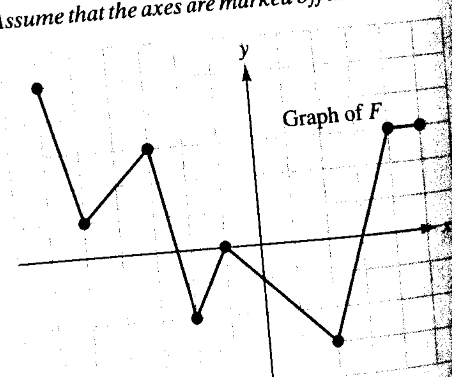


In Exercises 15 and 16, refer to the graph of the function F in the figure. (Assume that the axes are marked off in one-unit intervals.)

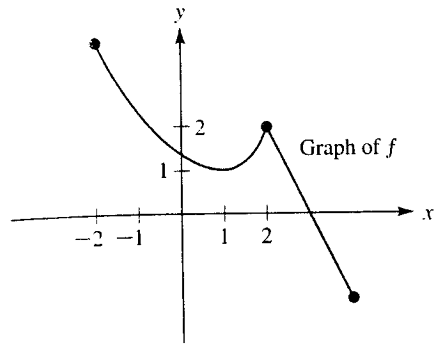


15. (a) Find $F(-5)$
(b) Find $F(2)$.
(c) Is $F(1)$ positive?
(d) For which value of x is $F(x) = -3$?
(e) Find $F(2) - F(-2)$.

16. (a) Find $F(4)$.
(b) Find $F(-1)$.
(c) Is $F(-4)$ positive?
(d) For which value of x is $F(x) = 5$?
(e) Find $F(5) - F(-3)$.

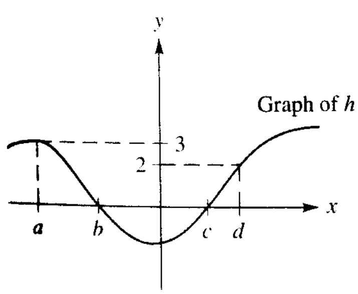
3.2

17. The following figure displays the graph of a function f.



(a) Is $f(0)$ positive or negative?
(b) Find $f(-2)$, $f(1)$, $f(2)$, and $f(3)$.
(c) Which is larger, $f(2)$ or $f(4)$?
(d) Find $f(4) - f(1)$.
(e) Find $|f(4) - f(1)|$.
(f) Write the domain and range of f using the interval notation $[a, b]$.

18. The following figure shows the graph of a function h.
(a) Find $h(a)$, $h(b)$, $h(c)$, and $h(d)$.
(b) Is $h(0)$ positive or negative?
(c) For which values of x does $h(x) = 0$?
(d) Which is larger, $h(b)$ or $h(0)$?
(e) As x increases from c to d, do the corresponding values of $h(x)$ increase or decrease?
(f) As x increases from a to b, do the corresponding values of $h(x)$ increase or decrease?

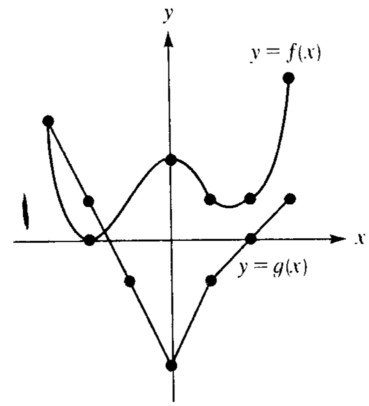


In Exercises 19 and 20, refer to the graphs of the functions f and g in the figure. Assume that the domain of each function is $[-3, 3]$ and that the axes are marked off in one-unit intervals.

19. (a) Which is larger, $f(-2)$ or $g(-2)$?
(b) Compute $f(0) - g(0)$.
(c) Which among the following three quantities is the smallest?

$$f(1) - g(1) \qquad f(2) - g(2) \qquad f(3) - g(3)$$

(d) For which value(s) of x does $g(x) = f(1)$?
(e) Is the number 4 in the range of f or in the range of g?



20. (a) For the interval $[0, 3]$, is the quantity $g(x) - f(x)$ positive or negative?
(b) For the interval $(-3, -2)$, is the quantity $g(x) - f(x)$ positive or negative?

In Exercises 21 and 22, set up a table and graph each function. [In Exercises 22(b) and 22(c), use a calculator to compute the square roots.]

21. (a) $y = |x|$
(b) $y = x^2$
(c) $y = x^3$

22. (a) $y = 1/x$
(b) $y = \sqrt{x}$
(c) $y = \sqrt{1 - x^2}$

In Exercises 23–30, sketch the graphs, as in Examples 5 and 6.

23. $A(x) = \begin{cases} x^3 & \text{if } -2 \le x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

24. $B(x) = \begin{cases} \sqrt{1 - x^2} & \text{if } -1 \le x < 1 \\ 1/x & \text{if } x \ge 1 \end{cases}$

25. $C(x) = \begin{cases} x^3 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

26. $y = \begin{cases} |x| & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$

27. (a) $y = \begin{cases} \sqrt{x} & \text{if } 0 \le x \le 1 \\ 1/x & \text{if } 1 < x < 2 \end{cases}$

(b) $y = \begin{cases} \sqrt{x} & \text{if } 0 \le x < 1 \\ 1/x & \text{if } 1 < x < 2 \end{cases}$

28. (a) $y = \begin{cases} x^3 & \text{if } x \le 0 \\ |x| & \text{if } x > 0 \end{cases}$

(b) $y = \begin{cases} |x| & \text{if } x \le 0 \\ x^3 & \text{if } x > 0 \end{cases}$

29. (a) $y = \begin{cases} \sqrt{1 - x^2} & \text{if } -1 \le x \le 0 \\ x^2 & \text{if } 0 < x \le 2 \end{cases}$

(b) $y = \begin{cases} \sqrt{1 - x^2} & \text{if } -1 \le x < 0 \\ x^2 & \text{if } 0 < x \le 2 \end{cases}$