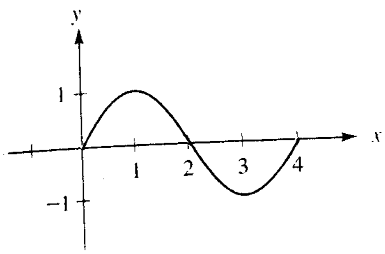
In Exercises 3–6, you are given functions with domain [0, 4]. Specify:
(a) the range of each function;
(b) the maximum value of the function;
(c) the minimum value of the function;
(d) interval(s) where the function is increasing; and
(e) interval(s) where the function is decreasing.

3.



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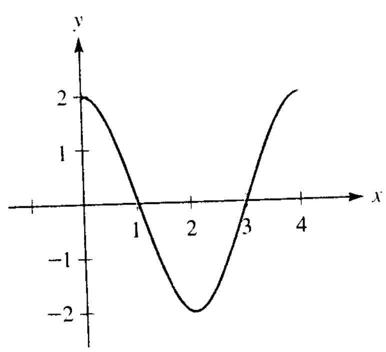

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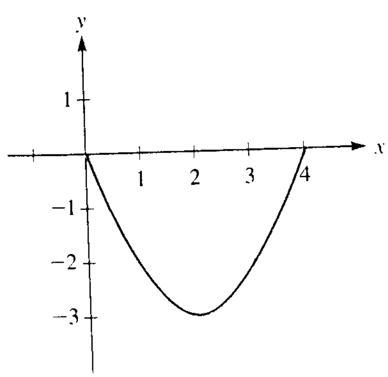
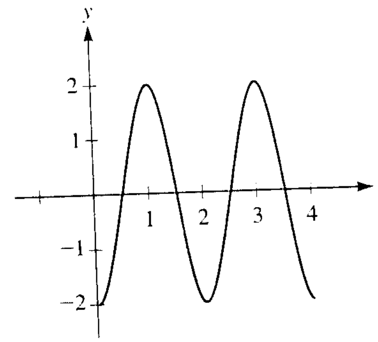
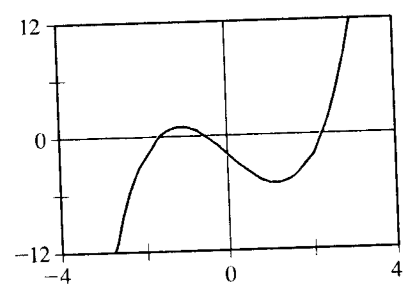


6.

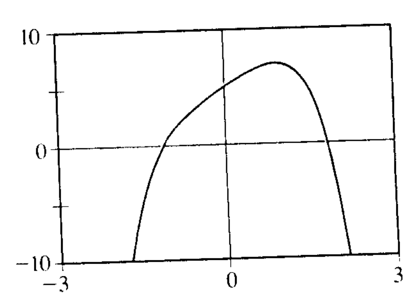


Ⓖ **7.** Assume that the accompanying viewing rectangle shows the essential features of the graph of $f(x) = x^3 - 4x - 2$. Use a graphing utility to estimate to the nearest hundredth the coordinates of the turning points. What are the corresponding estimates for the intervals where the function is increasing or decreasing?

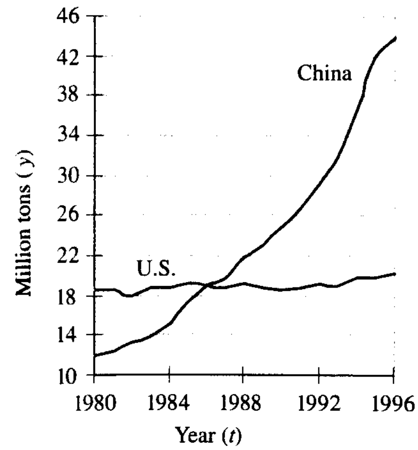


$f(x) = x^3 - 4x - 2$
[−4, 4, 2] by [−12, 12, 6]

8. Assume that the accompanying viewing rectangle shows the essential features of the graph of $f(x) = -x^4 + 3x + 5$. Use a graphing utility to estimate to the nearest hundredth the coordinates of the turning point. What is the corresponding estimate for the maximum value of the function? Estimate the intervals where the function is increasing or decreasing.



$f(x) = -x^4 + 3x + 5$
[−3, 3, 3] by [−10, 10, 5]



Consumption of red meat in United States and in China, 1980–1996

Source of data used to create graph: Worldwatch Institute

19. (a) Using values that you estimate from the graph, calculate $\Delta f/\Delta t$, the average rate of change of red meat consumption in the United States over the period 1980–1996. Express the answer as a fraction and as a decimal rounded to the nearest tenth.

(b) Use the data in the following table to obtain a more accurate value for $\Delta f/\Delta t$ over the period 1980–1996. Round the answer to two decimal places.

Year (t)	1980	1996
Consumption (y) in U.S. (million tons)	18.68	20.32

20. (a) Using values that you estimate from the graph, calculate $\Delta g/\Delta t$, the average rate of change of red meat consumption in China over the period 1980–1996.

(b) Use the data in the following table to obtain a more accurate value for $\Delta g/\Delta t$ over the period 1980–1996. Round the answer to two decimal places.

Year (t)	1980	1996
Consumption (y) in China (million tons)	11.90	43.83

21. Let $f(x) = 2x^2$.

(a) Find $\dfrac{f(x) - f(3)}{x - 3}$.

(b) Find $\dfrac{f(x) - f(a)}{x - a}$.

22. Let $f(x) = 4x^2$.

(a) Find $\dfrac{f(x) - f(-2)}{x + 2}$.

(b) Find $\dfrac{f(x) - f(a)}{x - a}$.

[illegible] 23 and 24, let $f(x) = x^2$. Find and simplify the in[illegible] quotient.

23. (a) $\dfrac{f(2 + h) - f(2)}{h}$ **(b)** $\dfrac{f(x + h) - f(x)}{h}$

24. (a) $\dfrac{f(1 + h) - f(1)}{h}$ **(b)** $\dfrac{f(t + h) - f(t)}{h}$

In Exercises 25–32:

(a) *Find the difference quotient* $\dfrac{f(x) - f(a)}{x - a}$ *for each function, as in Example 4.*

(b) *Find the difference quotient* $\dfrac{f(x + h) - f(x)}{h}$ *for each function, as in Example 5.*

25. $f(x) = 8x - 3$
26. $f(x) = -2x + 5$
27. $f(x) = x^2 - 2x + 4$
28. $f(x) = 2x^2 - x + 1$
29. $f(x) = 1/x$
30. $f(x) = -3/x^2$
31. $f(x) = 2x^3$
32. $f(x) = 1 - x^3$

Hint: For Exercises 31 and 32, you'll need to use difference-of-cubes factoring from intermediate algebra. See the inside back cover for the relevant formula. For a more detailed review of the topic, refer to Appendix B.4.

For Exercises 33–36, use the distance function $s(t) = 16t^2$ discussed on page 164 and in Example 6. Recall that this function relates the distance $s(t)$ and the time t for a freely falling object (neglecting air resistance). The time t is measured in seconds, with $t = 0$ corresponding to the instant that the object begins to fall; the distance $s(t)$ is in feet.

33. Find the average velocity $\Delta s/\Delta t$ over the time interval $1 \le t \le 2$.

34. (a) Find the average velocity over each of the following time intervals: [2, 3], [3, 4], and [2, 4].

(b) Let a, b, and c denote the three average velocities that you computed in part (a), in the order given. Is it true that the arithmetical average of a and b is c?

35. (a) Follow the method of Example 6(a) to find a general expression for the average velocity $\Delta s/\Delta t$ over the interval $[2, 2 + h]$.

(b) Complete a table similar to the one shown in the solution of Example 6(b); for the h-values in the left-hand column use 0.1, 0.01, 0.001, 0.0001, and 0.00001.

(c) Looking at your results in part (b), answer the following question. As h approaches zero, what value does the average velocity in the right-hand column seem to be approaching? This target value or limit is the *instantaneous* (as opposed to average) *velocity* of the object when $t = 2$ sec.

36. (a) After rereading Example 6, extend the results in part (b) of the example by completing the following table. Don't round your answers.

h (seconds)	0.01	0.001	0.0001	0.00001
Average velocity $\Delta s/\Delta t$ on interval $[1, 1 + h]$ (ft/second)				

(b) As h approaches zero, what value does the average velocity $\Delta s/\Delta t$ seem to be approaching? This target value or limit is called the *instantaneous velocity* of the object when $t = 1$ sec.

37. Suppose that the demand function for a certain item is given by $p(x) = \frac{24}{2^{x/100}}$, where x is the number of items that can be sold when the price of each item is $p(x)$ dollars. Compute $\Delta p/\Delta x$ over each of the intervals $0 \le x \le 100$ and $300 \le x \le 400$. Include units in your answers. Why does it make sense (economically) that the answers are negative?

38. Suppose that during the first few hours of a laboratory experiment, the temperature of a certain substance is closely approximated by the function $f(t) = t^3 - 6t^2 + 9t$, where t is measured in hours, with $t = 0$ corresponding to the instant the experiment begins, and $f(t)$ is the temperature (°F) of the substance after t hours.

(a) Find an expression for $\Delta f/\Delta t$. What are the units for $\Delta f/\Delta t$?

(b) Use the result in part (a) to complete the following table. Round the answers to four decimal places.

Interval	[0, 0.1]	[0, 0.01]	[0, 0.001]	[0, 0.0001]
$\Delta f/\Delta t$				

(c) In part (b), as the right-hand endpoint gets closer and closer to 0, what value does $\Delta f/\Delta t$ seem to be approaching? This target value tells us the rate at which the temperature is changing at the *instant* the experiment begins.

39. Complete the following table.

Function	$\lvert x \rvert$	x^2	x^3
Domain			
Range			
Turning point			
Maximum value			
Minimum value			
Interval(s) where increasing			
Interval(s) where decreasing			

40. Set up and complete a table like the one in Exercise 39 for the three functions $1/x$, $\sqrt{x}$, and $\sqrt{1 - x^2}$.

B

41. Let $f(x) = 1/x$. Find a number b so that the average rate of change of f on the interval $[1, b]$ is $-1/5$.

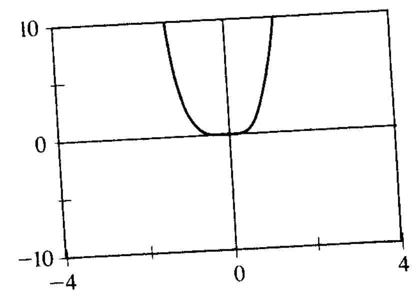
42. Let $f(x) = \sqrt{x}$. Find a number b so that the average rate of change of f on the interval $[1, b]$ is $1/7$.

43. Let $f(x) = ax^2 + bx + c$. Show that
$$\frac{f(x + h) - f(x)}{h} = 2ax + ah + b.$$

44. Find a number a between 0 and 1 so that the average rate of change of $f(x) = x^2$ on the interval $[a, 1/a]$ is $10a$.

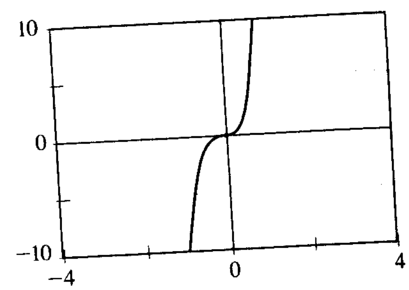
Ⓖ *In Exercises 45 and 46 (as opposed to Exercises 7 and 8), the viewing rectangles do not show all of the essential features of the graphs. In each case, use a graphing utility and experiment with different viewing rectangles to determine approximate x- and y-coordinates for all turning points. What are the corresponding intervals of increase and decrease for the function?*

45.



$f(x) = 4x^4 + x^3$
$[-4, 4, 2]$ by $[-10, 10, 5]$

46.



$g(x) = x^4 + 12x^3 + x^2$
$[-4, 4, 2]$ by $[-10, 10, 5]$

47. Let $h(x) = \sqrt{x}$. Two functions f and g are defined in terms of h as follows:
$$f(x) = \frac{h(x) - h(2)}{x - 2} \qquad g(x) = \frac{1}{h(x) + h(2)}$$

Ⓖ **(a)** Using a graphing utility, graph the two functions f and g in the same viewing screen. What do you observe?

(b) Use algebra to explain the result in part (a). Th… either derive or verify the identity $f(x) = g(x)$… there any positive values of x for which this… does not hold?

48. Consider the function f defined by
$$f(x) = x^2 + \frac{2}{x^2} \quad (x > 0)$$

(a) Complete the table. (Round the results to f… mal places.)

x	1	1.05	1.10	1.15	1.2…
f(x)					

(b) Whicl… the cc…

Ⓖ (c) Use a… use th… put f(… part (…

…geometrica… formulas. —Da… (1862–1943)

y = G(x)

Figure 1