① $f(x) = 5\ln x + 8\sqrt{x}$

$f'(x) = 5\left(\frac{1}{x}\right) + 8\left(\frac{1}{2\sqrt{x}}\right) = \frac{5}{x} + \frac{4}{\sqrt{x}}$, so $\underline{m} = f'(1) = \underline{9}$

WHEN $x = 1$, $\underline{y} = f(1) = 5(0) + 8(1) = \underline{8}$; SO THE TANGENT LINE AT $(1, f(1))$ IS GIVEN BY $\boxed{y - 8 = 9(x-1)}$ OR $\boxed{y = 9x - 1}$

② $f(x) = x^2 (\ln x)^2$

$\underline{f'(x)} = x^2 \cdot \left(2\ln x \cdot \frac{1}{x}\right) + 2x(\ln x)^2 = 2x\ln x + 2x(\ln x)^2 = \underline{2x\ln x\,(1 + \ln x)}$,

SO $f'(x) = 0$ WHEN $\cancel{x = 0}$ (NOT IN THE DOMAIN), $\ln x = 0$, OR $\ln x = -1$

$\underline{x = 1}$, $\underline{x = e^{-1}}$

f': $+$ on $(0, \frac{1}{e})$, $-$ on $(\frac{1}{e}, 1)$, $+$ on $(1, \infty)$

$f'\left(\frac{1}{e^2}\right) = \frac{4}{e^2}$ $\quad f'(e^{-1/2}) = -\frac{1}{2\sqrt{e}}$ $\quad f'(e) = 4e$

$$\boxed{\begin{array}{l} f\left(\frac{1}{e}\right) = \frac{1}{e^2} \text{ IS A REL. MAX.} \\ f(1) = 0 \text{ IS A REL. MIN.} \end{array}}$$

③ $y = \ln(e^{5x} + 2^x)$ $\qquad y' = \boxed{\dfrac{e^{5x} \cdot 5 + 2^x \ln 2}{e^{5x} + 2^x}}$

④ $y = \log_2(\cos 3x + \tan\sqrt{x})$

$$y' = \boxed{\dfrac{-\sin 3x \cdot 3 + \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}}{(\cos 3x + \tan\sqrt{x})(\ln 2)}}$$

⑤ $y = \ln(\ln x)$ $\qquad y' = \dfrac{\frac{1}{x}}{\ln x} = \boxed{\dfrac{1}{x\ln x}}$

⑥ $y = \dfrac{(\ln 8x)^3}{x^2}$ $\qquad y' = \boxed{\dfrac{x^2\left(3(\ln 8x)^2 \cdot \frac{8}{8x}\right) - (\ln 8x)^3 \cdot 2x}{(x^2)^2}}$

OR $y = (\ln 8x)^3 x^{-2}$

SO $y' = \boxed{(\ln 8x)^3(-2x^{-3}) + \left(3(\ln 8x)^2 \cdot \frac{8}{8x}\right)x^{-2}}$

(7) $Y = \ln(\sin^5 2x) = 5\ln(\sin 2x)$

$$Y' = 5 \cdot \frac{\cos 2x \cdot 2}{\sin 2x} = \boxed{10 \cot 2x}$$

(8) $Y = \cot(5^{x^2})$ $\quad Y' = \boxed{-\csc^2(5^{x^2}) \cdot (5^{x^2} \cdot \ln 5 \cdot 2x)}$

(9) $Y = \ln(x + \sqrt{4+x^2})$

$$Y' = \frac{1 + \frac{1}{\cancel{2}}(4+x^2)^{-1/2} \cdot \cancel{2}x}{x + \sqrt{4+x^2}} = \frac{1 + \frac{x}{\sqrt{4+x^2}}}{x + \sqrt{4+x^2}} \cdot \frac{\sqrt{4+x^2}}{\sqrt{4+x^2}}$$

$$= \frac{\cancel{\sqrt{4+x^2} + x}}{(\cancel{x + \sqrt{4+x^2}})(\sqrt{4+x^2})} = \boxed{\frac{1}{\sqrt{4+x^2}}}$$

(10) $Y = \ln(\sec x)$ $\quad Y' = \frac{\sec x \tan x}{\sec x} = \boxed{\tan x}$

(11) $Y = \ln(\sec x + \tan x)$

$$Y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x\,(\cancel{\tan x + \sec x})}{\cancel{\sec x + \tan x}} = \boxed{\sec x}$$

(12) $Y = \ln\left(\frac{\sqrt[4]{x}\,(e^{2x}+7)^5}{3^{\sqrt{x}}\,(5x+8)^9}\right) = \ln\left(x^{1/4}(e^{2x}+7)^5\right) - \ln\left(3^{\sqrt{x}}(5x+8)^9\right)$

$$= \left[\ln x^{1/4} + \ln(e^{2x}+7)^5\right] - \left[\ln 3^{\sqrt{x}} + \ln(5x+8)^9\right]$$

$$= \frac{1}{4}\ln x + 5\ln(e^{2x}+7) - \sqrt{x}\ln 3 - 9\ln(5x+8),$$

so $Y' = \boxed{\frac{1}{4} \cdot \frac{1}{x} + 5 \cdot \frac{e^{2x} \cdot 2}{e^{2x}+7} - \frac{1}{2\sqrt{x}} \cdot \ln 3 - 9 \cdot \frac{5}{5x+8}}$

(13) $Y = (e^{4x} + x^2)^{5x}$

1) $\ln Y = \ln(e^{4x}+x^2)^{5x} = 5x\ln(e^{4x}+x^2)$

2) $\frac{Y'}{Y} = 5x \cdot \frac{e^{4x} \cdot 4 + 2x}{e^{4x}+x^2} + 5\ln(e^{4x}+x^2)$

3) $Y' = \boxed{(e^{4x}+x^2)^{5x}\left[5x \cdot \frac{e^{4x} \cdot 4 + 2x}{e^{4x}+x^2} + 5\ln(e^{4x}+x^2)\right]}$

(14) $Y = (x^4 + 3^x)^{\log_5 x}$

1) $\ln Y = \ln (x^4 + 3^x)^{\log_5 x} = (\log_5 x)(\ln(x^4 + 3^x))$

2) $\dfrac{Y'}{Y} = (\log_5 x) \cdot \dfrac{4x^3 + 3^x \ln 3}{x^4 + 3^x} + \dfrac{1}{x \ln 5} \cdot \ln(x^4 + 3^x)$

3) $\boxed{Y' = (x^4 + 3^x)^{\log_5 x}\left[(\log_5 x) \cdot \dfrac{4x^3 + 3^x \ln 3}{x^4 + 3^x} + \dfrac{1}{x \ln 5} \cdot \ln(x^4 + 3^x)\right]}$

(15) $Y = 3^{1/x} x^{\sqrt{x}}$

1) $\ln Y = \ln(3^{1/x} x^{\sqrt{x}}) = \ln 3^{1/x} + \ln x^{\sqrt{x}} = \frac{1}{x} \cdot \ln 3 + \sqrt{x} \ln x$

2) $\dfrac{Y'}{Y} = \left(-\dfrac{1}{x^2}\right) \cdot \ln 3 + \sqrt{x} \cdot \dfrac{1}{x} + \dfrac{1}{2\sqrt{x}} \cdot \ln x$

3) $\boxed{Y' = (3^{1/x} x^{\sqrt{x}})\left[-\dfrac{\ln 3}{x^2} + \dfrac{1}{\sqrt{x}} + \dfrac{\ln x}{2\sqrt{x}}\right]}$

OR Let $T = x^{\sqrt{x}}$, so

1) $\ln T = \ln x^{\sqrt{x}} = \sqrt{x} \ln x$

2) $\dfrac{T'}{T} = \sqrt{x} \cdot \dfrac{1}{x} + \dfrac{1}{2\sqrt{x}} \cdot \ln x = \dfrac{1}{\sqrt{x}} + \dfrac{\ln x}{2\sqrt{x}}$

3) $T' = x^{\sqrt{x}}\left[\dfrac{1}{\sqrt{x}} + \dfrac{\ln x}{2\sqrt{x}}\right]$

By the product rule,

$\boxed{Y' = 3^{1/x}\left(x^{\sqrt{x}}\left[\dfrac{1}{\sqrt{x}} + \dfrac{\ln x}{2\sqrt{x}}\right]\right) + \left(3^{1/x} \cdot \ln 3 \cdot \left(-\dfrac{1}{x^2}\right)\right) x^{\sqrt{x}}}$

(16) $f(x) = \ln x$, so

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h} = \lim_{h \to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\ln\left(1 + \frac{h}{x}\right)}{h} \longleftarrow \text{Let } u = \frac{h}{x}, \text{ so } h = ux \text{ and } u \to 0 \text{ as } h \to 0.$$

$$= \lim_{u \to 0} \frac{\ln(1+u)}{ux}$$

$$= \lim_{u \to 0} \frac{1}{u} \ln(1+u) \cdot \frac{1}{x} = \lim_{u \to 0} \ln(1+u)^{\frac{1}{u}} \cdot \frac{1}{x}$$

$$= \left[\ln\left(\lim_{u \to 0} (1+u)^{\frac{1}{u}}\right)\right] \cdot \frac{1}{x} = [\ln e] \cdot \frac{1}{x} = 1 \cdot \frac{1}{x} = \boxed{\frac{1}{x}}$$