MATH 16B

SOLUTIONS – CH. 4

P. 315 – (127) $Y = Ce^{KT} = 500e^{KT}$

WHEN $T = 40$, $Y = 300$; SO $500e^{40K} = 300$, $e^{40K} = \frac{3}{5}$, $40K = \ln\frac{3}{5}$,

$K = \frac{1}{40}\ln\frac{3}{5}$ SO $\boxed{Y = 500e^{\left(\frac{1}{40}\ln\frac{3}{5}\right)T}}$

OR $Y = 500\left(e^{\ln\frac{3}{5}}\right)^{\frac{T}{40}}$ SO $\boxed{Y = 500\left(\frac{3}{5}\right)^{\frac{T}{40}}}$

(129) $Y = Ce^{KT} = 50e^{KT}$

WHEN $T = 7$, $Y = 42.031$; SO $50e^{7K} = 42.031$, $e^{7K} = .84062$,

$7K = \ln .84062$, $K = \frac{1}{7}\ln .84062$

SO $Y = 50e^{\left(\frac{1}{7}\ln .84062\right)T} = 50\left(e^{\ln .84062}\right)^{T/7} = 50(.84062)^{T/7}$

WHEN $Y = \frac{1}{2}(50)$, $\cancel{50}(.84062)^{T/7} = \frac{1}{2}(\cancel{50})$ SO $(.84062)^{T/7} = \frac{1}{2}$,

$\ln(.84062)^{T/7} = \ln .5$, $\frac{T}{7}\ln .84062 = \ln .5$, $T = \boxed{\frac{7\ln .5}{\ln .84062}\text{ YR}} \approx 27.9$ YR

P. A33 – (37) $T = S + Ce^{KT} = 90 + Ce^{KT}$

1) WHEN $t = 0$, $T = 1500$; SO $1500 = 90 + C\cdot 1$ SO $C = 1410$ AND

$T = 90 + 1410e^{Kt}$

2) WHEN $t = 1$, $T = 1120$; SO $1120 = 90 + 1410e^{K}$, $1030 = 1410e^{K}$, $e^{K} = \frac{103}{141}$,

$T = 90 + 1410(e^{K})^{t} = 90 + 1410\left(\frac{103}{141}\right)^{t}$

3) WHEN $t = 5$, $T = \boxed{90 + 1410\left(\frac{103}{141}\right)^{5}\text{ DEGREES}} \approx 383.3°$

(38) $T = S + Ce^{Kt} = 70 + Ce^{Kt}$

1) WHEN $t = 0$, $T = 350$; SO $350 = 70 + C\cdot 1$ SO $C = 280$ AND

$T = 70 + 280e^{Kt}$

2) WHEN $t = 45$, $T = 150$; SO $150 = 70 + 280e^{45K}$, $80 = 280e^{45K}$,

$e^{45K} = \frac{80}{280} = \frac{2}{7}$, $45K = \ln\frac{2}{7}$, $K = \frac{1}{45}\ln\frac{2}{7}$ SO

$T = 70 + 280e^{\left(\frac{1}{45}\ln\frac{2}{7}\right)t} = 70 + 280\left(e^{\ln\frac{2}{7}}\right)^{t/45} = 70 + 280\left(\frac{2}{7}\right)^{t/45}$

3) WHEN $T = 80$, $80 = 70 + 280\left(\frac{2}{7}\right)^{t/45}$, $10 = 280\left(\frac{2}{7}\right)^{t/45}$, $\left(\frac{2}{7}\right)^{t/45} = \frac{1}{28}$,

$\ln\left(\frac{2}{7}\right)^{t/45} = \ln\frac{1}{28}$, $\frac{t}{45}\ln\frac{2}{7} = \ln\frac{1}{28}$, $t = \boxed{\frac{45\ln\frac{1}{28}}{\ln\frac{2}{7}}\text{ MIN}} \approx 120$ MIN

MATH 16B SOLUTIONS – CH. 4 AND SEC. 5.1

4.6 – (6) $Y = Ce^{KT}$

1) WHEN $T = 3$, $Y = \frac{1}{2}$; SO $Ce^{3K} = \frac{1}{2}$

2) WHEN $T = 4$, $Y = 5$; SO $Ce^{4K} = 5$

THEN $\frac{\cancel{C}e^{4K}}{\cancel{C}e^{3K}} = \frac{5}{1/2} \Rightarrow e^{K} = 10$ AND $K = \text{LN } 10$;

AND $C(e^{K})^4 = 5 \Rightarrow C(10)^4 = 5 \Rightarrow C = \frac{5}{10^4} = \frac{1}{2{,}000}$

THEREFORE $Y = \boxed{\frac{1}{2000} e^{(\text{LN } 10)T}} = \boxed{\frac{1}{2000}(10^T)} = \frac{5}{10^4}(10^T) = \boxed{5(10^{T-4})}$

P. 315 – (130) $Y = Ce^{KT} = .5e^{KT}$ WHEN $T = 5.2$, $Y = \frac{1}{2}(.5)$; SO $\cancel{.5}e^{5.2K} = \frac{1}{2}(\cancel{.5}) \Rightarrow$

$e^{5.2K} = \frac{1}{2} \Rightarrow 5.2K = \text{LN } .5 \Rightarrow K = \frac{1}{5.2}\text{LN } .5$ AND $Y = .5e^{(\frac{1}{5.2}\text{LN } .5)T}$

THEN $Y = .1 \Rightarrow .5e^{(\frac{1}{5.2}\text{LN } .5)T} = .1 \Rightarrow e^{(\frac{1}{5.2}\text{LN } .5)T} = .2 \Rightarrow$

$\left(\frac{1}{5.2}\text{LN } .5\right)T = \text{LN } .2 \Rightarrow T = \boxed{\frac{5.2\,\text{LN } .2}{\text{LN } .5}\text{ YRS}} \approx 12\text{ YRS}$

5.1 – (34) $\int (x^2 - 2x + 3)\,dx = \boxed{\frac{x^3}{3} - x^2 + 3x + C}$

(62) $f''(x) = x^{-3/2}$, $f'(1) = 2$, $f(9) = -4$

1) $f'(x) = \int x^{-3/2}\,dx = -2x^{-1/2} + C$ $\quad f'(1) = -2 + C = 2$ SO $C = 4$

$f'(x) = -2x^{-1/2} + 4$

2) $f(x) = \int (-2x^{-1/2} + 4)\,dx = -4x^{1/2} + 4x + D$

$f(9) = -4 \cdot 3 + 4 \cdot 9 + D = -4$ SO $D = -28$

$\boxed{f(x) = -4x^{1/2} + 4x - 28}$

(78) $a(t) = -32$; $v(0) = 16$, $s(0) = 64$

1) $v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C = -32t + 16$ SINCE $C = v(0) = 16$

2) $s(t) = \int v(t)\,dt = \int (-32t + 16)\,dt = -16t^2 + 16t + D = -16t^2 + 16t + 64$

SINCE $D = s(0) = 64$

a) THE BAG HITS THE GROUND WHEN $s(t) = 0$:

$-16t^2 + 16t + 64 = 0$

$t^2 - t - 4 = 0$ $\quad t = \frac{1 \pm \sqrt{17}}{2}$ SO $t = \boxed{\frac{1 + \sqrt{17}}{2}\text{ sec}}$ (SINCE $t > 0$)

b) WHEN $t = \frac{1 + \sqrt{17}}{2}$,

$v(t) = -32\left(\frac{1 + \sqrt{17}}{2}\right) + 16 = \boxed{-16\sqrt{17}\text{ FT/sec}}$