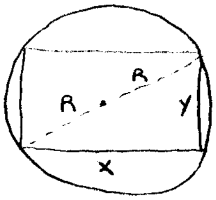
MATH 12

Sec. 4.5 – Solutions

(33)



1) MAXIMIZE $A = XY$

2) $X^2 + Y^2 = (2R)^2 = 4R^2$, so $Y^2 = 4R^2 - X^2$ and $Y = \sqrt{4R^2 - X^2}$.

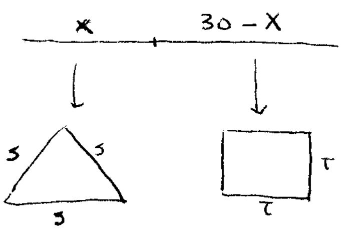
3) Then $A = X\sqrt{4R^2 - X^2}$, so we can maximize

$$A^2 = X^2(4R^2 - X^2) = 4R^2X^2 - X^4.$$

Let $\boxed{T = X^2}$, so $A^2 = 4R^2T - T^2$.

4) A^2 has a max. when $T = -\frac{b}{2a} = -\frac{4R^2}{2(-1)} = 2R^2$, so $X^2 = 2R^2$ and $X = \sqrt{2}R$. Then $A = (\sqrt{2}R)\sqrt{4R^2 - 2R^2} = (\sqrt{2}R)\sqrt{2R^2} = (\sqrt{2}R)(\sqrt{2}R) = \boxed{2R^2}$ is the maximum area for the rectangle.

(46)



1) Minimize $A = \frac{\sqrt{3}}{4}s^2 + T^2$ (see 4.4, # 13)

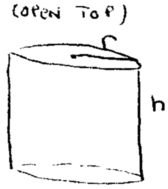
2) $3s = x$ and $4T = 30 - x$, so $s = \frac{x}{3}$ and $T = \frac{1}{4}(30 - x)$.

3) $$A = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \left(\frac{1}{4}(30 - x)\right)^2 = \frac{\sqrt{3}}{4}\left(\frac{x^2}{9}\right) + \frac{1}{16}(900 - 60x + x^2)$$
$$= \frac{\sqrt{3}}{36}x^2 + \frac{225}{4} - \frac{15}{4}x + \frac{1}{16}x^2 = \left(\frac{\sqrt{3}}{36} + \frac{1}{16}\right)x^2 - \frac{15}{4}x + \frac{225}{4}$$
$$= \left(\frac{4\sqrt{3} + 9}{144}\right)x^2 - \frac{15}{4}x + \frac{225}{4}$$

4) A has a min. when $x = -\frac{b}{2a} = -\frac{-\frac{15}{4}}{2\left(\frac{4\sqrt{3}+9}{144}\right)} = \frac{\frac{15}{4}}{\frac{4\sqrt{3}+9}{72}} = \frac{15}{4} \cdot \frac{72}{4\sqrt{3}+9} = \boxed{\frac{270}{4\sqrt{3}+9} \text{ in}}$

(47a)

(OPEN TOP)

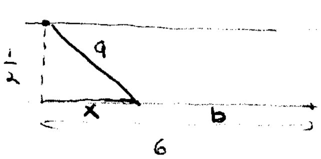


1) Minimize $S = \underset{\text{(BOTTOM)}}{\pi r^2} + \underset{\text{(SIDE)}}{2\pi r h}$

2) $V = \pi r^2 h = 500$, so $h = \frac{500}{\pi r^2}$

3) $\boxed{S} = \pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{500}{\cancel{\pi} r^{\cancel{2}}}\right) = \boxed{\pi r^2 + \frac{1000}{r}}$

(48a)



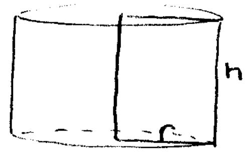
1) $C = 8(5280)a + 6(5280)b$ (since 1 mi = 5280 ft)

2) $a^2 = x^2 + \left(\frac{1}{2}\right)^2 = x^2 + \frac{1}{4}$, so $a = \sqrt{x^2 + \frac{1}{4}}$

and $x + b = 6$ so $b = 6 - x$

3) $\boxed{C = 5280\left[8\sqrt{x^2 + \frac{1}{4}} + 6(6 - x)\right]}$

4.5 – 50a 1)

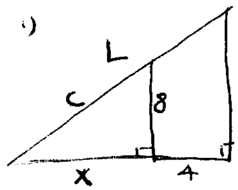


2) MAXIMIZE $V = \pi r^2 h$

3) $2r + 2h = 36$, so $r + h = 18$ AND $h = 18 - r$

4) $\boxed{V = \pi r^2 (18 - r)}$ OR $\boxed{V = \pi (18r^2 - r^3)}$

52a 1)



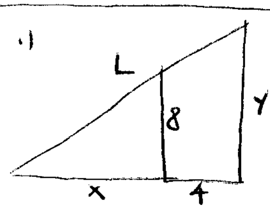
2) MINIMIZE L, WHERE $\frac{L}{x+4} = \frac{c}{x}$ SO $L = \frac{c}{x}(x+4)$

(BY SIMILAR TRIANGLES)

3) $c^2 = x^2 + 64$, SO $c = \sqrt{x^2+64}$ BY THE PYTHAGOREAN TH.

4) $\boxed{L = \frac{\sqrt{x^2+64}}{x}(x+4)}$ OR $\boxed{L = \frac{x+4}{x}\left(\sqrt{x^2+64}\right)}$

OR 1)

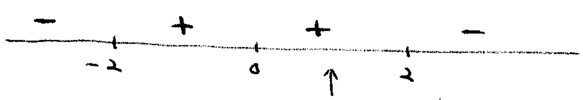


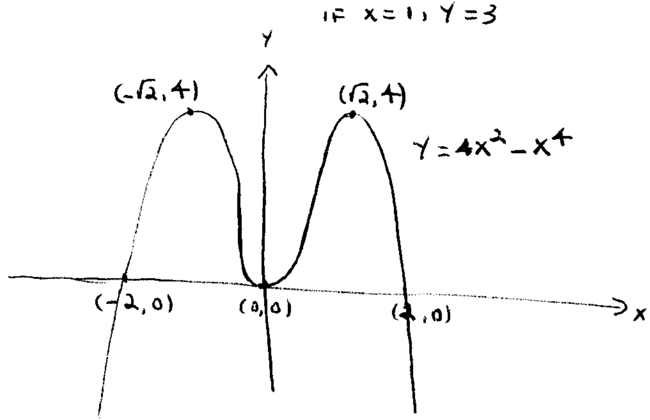
2) MINIMIZE $L^2 = (x+4)^2 + Y^2$

3) BY SIMILAR TRIANGLES, $\frac{Y}{x+4} = \frac{8}{x}$ SO $Y = \frac{8(x+4)}{x}$

4) $L^2 = (x+4)^2 + \frac{64(x+4)^2}{x^2}$ SO $\boxed{L^2 = (x+4)^2\left(1 + \frac{64}{x^2}\right)}$

4.6 – 60 a) $Y = 4x^2 - x^4 = x^2(4 - x^2) = x^2(2-x)(2+x)$





b) IF WE LET $T = x^2$, $Y = 4T - T^2$ SO Y HAS A MAX. WHEN

$T = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$: $x^2 = 2$ SO $x = \pm\sqrt{2}$

AND $Y = 8 - 4 = 4$

TURNING POINTS : $\boxed{(-\sqrt{2}, 4) \text{ AND } (\sqrt{2}, 4)}$

OR $Y = 4x^2 - x^4 = -(x^4 - 4x^2 + 4) + 4 = -(x^2-2)^2 + 4$,

SO Y HAS A MAX. VALUE OF 4 WHEN $x^2 = 2$ SO $x = \pm\sqrt{2}$

TURNING POINTS : $\boxed{(-\sqrt{2}, 4) \text{ AND } (\sqrt{2}, 4)}$

(28) $Y = \dfrac{2X}{(X+1)^2}$

1) VERTICAL ASYMPTOTE: $\underline{X = -1}$

HORIZONTAL ASYMPTOTE: $\underline{Y = 0}$

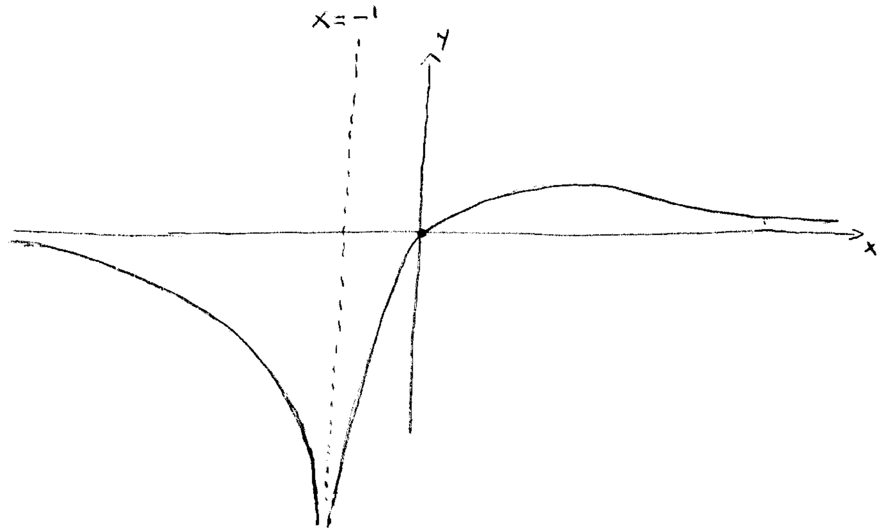
2) Y-INT.: $X=0$ GIVES $Y = \underline{0}$

X-INT.: $Y=0$ GIVES $X = \underline{0}$

3) $-$ (×) -1 $-$ 0 $+$

IF $X = 1$, $Y = \frac{2}{4} = \frac{1}{2}$

4) THE GRAPH INTERSECTS $Y=0$ WHERE $\dfrac{2X}{(X+1)^2} = 0$, SO $\underline{X=0}$



(42) $Y = \dfrac{2X^2 - 3X - 2}{X^2 - 3X - 4} = \dfrac{(2X+1)(X-2)}{(X-4)(X+1)}$

1) VERTICAL ASYMPTOTES: $\underline{X = -1 \text{ AND } X = 4}$

HORIZONTAL ASYMPTOTE: $\underline{Y = 2}$

2) Y-INT.: $X=0$ GIVES $Y = \frac{-2}{-4} = \underline{\frac{1}{2}}$

X-INT.: $Y=0$ GIVES $(2X+1)(X-2)=0$

$X = \underline{-\frac{1}{2}}$ OR $X = \underline{2}$

3) $+$ (×) -1 $-$ $-\frac{1}{2}$ $+$ 2 $-$ (×) 4 $+$

IF $X=0$, $Y = \frac{1}{2}$

4) THE GRAPH INTERSECTS $Y=2$ WHEN $\dfrac{2X^2-3X-2}{X^2-3X-4} = 2$:

$2X^2 - 3X - 2 = 2(X^2 - 3X - 4)$ SO $\cancel{2X^2} - 3X - 2 = \cancel{2X^2} - 6X - 8$ SO $3X = -6$ AND $\underline{X = -2}$