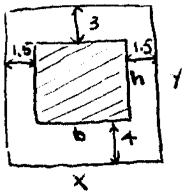
MATH 12

Sec. 4.7 – SOLUTIONS

(37a)

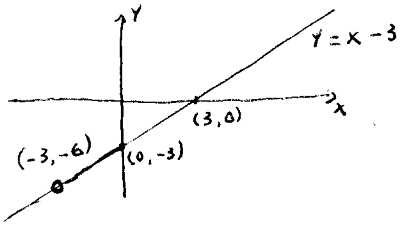


1) $A = bh$ WHERE $x = b + 3$ AND $y = h + 7$, SO $\underline{A = (x-3)(y-7)}$

2) $xy = 500$, SO $\underline{y = \frac{500}{x}}$ AND

3) $A = (x-3)\left(\frac{500}{x} - 7\right) = 500 - 7x - \frac{1500}{x} + 21 = \boxed{521 - 7x - \frac{1500}{x}}$

(44) ⓐ $y = \frac{x^2 - 9}{x + 3} = \frac{(x-3)(x+3)}{x+3} = x - 3$ (FOR $x \neq -3$)



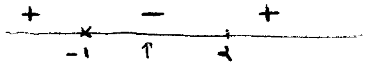
(THE GRAPH IS THE LINE $y = x - 3$ WITH THE POINT $(-3,-6)$ DELETED.)

ⓑ $y = \frac{x^2 - 5x + 6}{x^2 - 2x - 3} = \frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x+1)} = \frac{x-2}{x+1}$ (FOR $x \neq 3$)

1) VERTICAL ASYMP. : $\underline{x = -1}$
HORIZONTAL ASYMP. : $\underline{y = 1}$

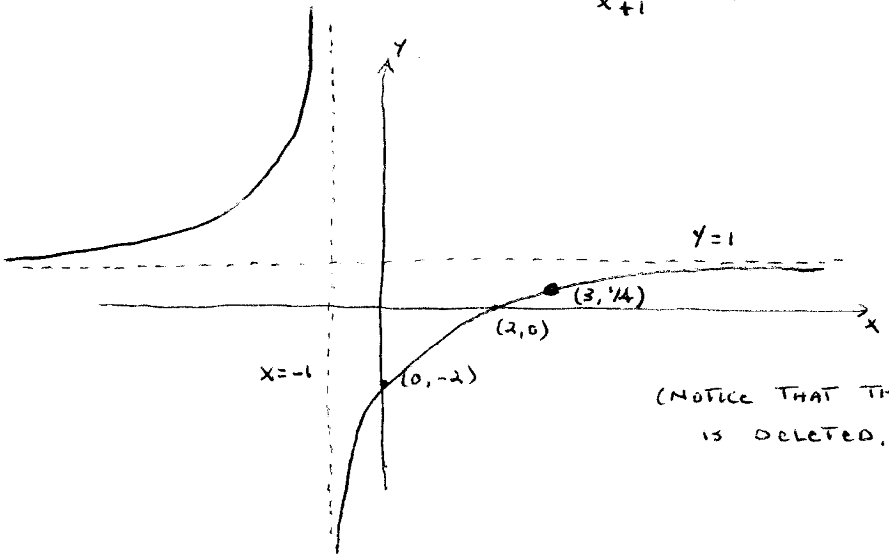
2) $\underline{y\text{-INT.}}$: $x = 0$ GIVES $y = \underline{-2}$
$\underline{x\text{-INT.}}$: $y = 0$ GIVES $x = \underline{2}$

3)



IF $x = 0$, $y = -2$

4) THE GRAPH INTERSECTS THE HA WHERE $\frac{x-2}{x+1} = 1$, SO $x - 2 = x + 1$ AND $-2 = 1$
(NO SOLUTION)



(NOTICE THAT THE POINT $(3, \frac{1}{4})$ IS DELETED.)

MATH 12

Sec. 4.7 AND 3.6 -- SOLUTIONS

4.7 - (50) $F(x) = \dfrac{x^2}{x+2}$

$$\begin{array}{r} x-2 \\ x+2\,\overline{)\,x^2} \\ x^2+2x \\ \hline -2x \\ -2x-4 \\ \hline 4 \end{array}$$

so $F(x) = x-2+\dfrac{4}{x+2}$

1) VERTICAL ASYMP. : $x=-2$

SLANTED ASYMP. : $y = x-2$

2) Y-INT. ; $x=0$ GIVES $y=0$

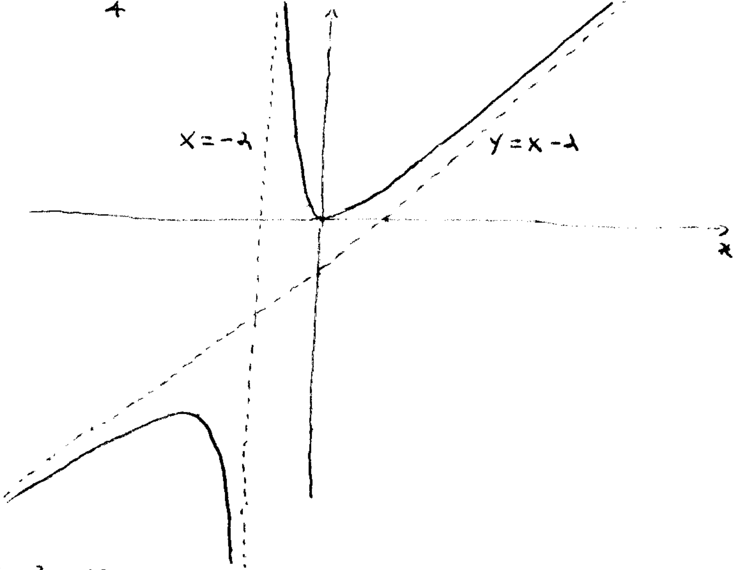
X-INT. : $y=0$ GIVES $x^2=0$, so $x=0$

3)
$$\begin{array}{ccccc} - & & + & & + \\ \hline & -2 & & 0 & \uparrow \end{array}$$
IF $x=1$, $y=\frac{1}{3}$

4) THE GRAPH INTERSECTS THE SLANTED ASYMPTOTE WHERE $F(x) = x-2$, so

$\cancel{x-2} + \dfrac{4}{x+2} = \cancel{x-2}$ GIVES $\dfrac{4}{x+2} = 0$

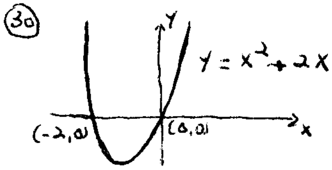
so $4=0$ (NO SOLUTION)



3.6 - (21) a) $f^{-1}(2x+3)=5 \Rightarrow f(f^{-1}(2x+3)) = f(5) \Rightarrow 2x+3 = 13 \Rightarrow 2x = 10 \Rightarrow \boxed{x=5}$

b) $f(1-2x) = -4 \Rightarrow f^{-1}(f(1-2x)) = f^{-1}(4) \Rightarrow 1-2x = -5 \Rightarrow -2x=-6 \Rightarrow \boxed{x=3}$

(30)



THE FUNCTION IS NOT 1-1, SINCE THERE IS A HORIZONTAL LINE WHICH INTERSECTS THE GRAPH MORE THAN ONCE.

(36) $f(x) = (x-3)^3-1$ [NOTICE THAT THE GRAPH OF f IS THE GRAPH OF $Y=X^3$ SHIFTED 3 UNITS TO THE RIGHT AND 1 UNIT DOWN.]

a) DOMAIN : $\boxed{(-\infty,\infty)}$

RANGE ; THE EQUATION $Y=(x-3)^3-1$ GIVES $Y+1=(x-3)^3$, $\sqrt[3]{Y+1} = x-3$, $x=\sqrt[3]{Y+1}+3$, SINCE THIS SOLUTION IS DEFINED FOR ANY VALUE OF Y, THE RANGE IS $\boxed{(-\infty,\infty)}$.

b) SINCE THE EQUATION $Y=(x-3)^3-1$ HAS A UNIQUE SOL. FOR X FOR ANY GIVEN VALUE OF Y, f^{-1} EXISTS. [$\boxed{\text{OR}}$ SHOW THAT f IS 1-1: IF $f(a)=f(b)$, THEN $(a-3)^3-1=(b-3)^3-1$, so $(a-3)^3=(b-3)^3$, $a-3=b-3$, AND THEREFORE $a=b$.]

DOMAIN OF f^{-1} = RANGE OF f = $(-\infty,\infty)$, AND

RANGE OF f^{-1} = DOMAIN OF f = $(-\infty,\infty)$

d) BY PART a), $Y=(x-3)^3-1$ GIVES $x=\sqrt[3]{Y+1}+3$, so $f^{-1}(Y)=\sqrt[3]{Y+1}+3$

AND $\boxed{f^{-1}(x)=\sqrt[3]{x+1}+3}$

(39) a) $f(x)=\sqrt{x}$

1) $Y=\sqrt{x}$

2) $x=Y^2$

3) $f^{-1}(Y)=Y^2$, so $\boxed{f^{-1}(x)=x^2}$

DOMAIN OF f^{-1} = RANGE OF f = $\boxed{[0,\infty)}$