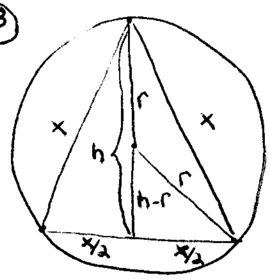
MATH 12

SEC. 4.4 AND 4.5 – SOLUTIONS

4.4 – 43



1) $A = \pi r^2$

2) $h^2 + \left(\frac{x}{2}\right)^2 = x^2$ BY THE PYTHAGOREAN TH., SO

$h^2 + \frac{x^2}{4} = x^2$, $h^2 = x^2 - \frac{x^2}{4} = \frac{4x^2}{4} - \frac{x^2}{4} = \frac{3x^2}{4}$, AND $h = \frac{\sqrt{3}}{2}x$.

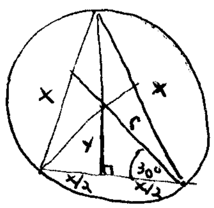
$(h-r)^2 + \left(\frac{x}{2}\right)^2 = r^2$ BY THE PYTHAGOREAN TH., SO

$h^2 - 2rh + \cancel{r^2} + \frac{x^2}{4} = \cancel{r^2} \implies \frac{3x^2}{4} - 2r\left(\frac{\sqrt{3}}{2}x\right) + \frac{x^2}{4} = 0 \implies$

$x^2 - \sqrt{3}rx = 0 \implies x(x - \sqrt{3}r) = 0 \implies \cancel{x=0}$ OR $x = \sqrt{3}r$.

THEN $r = \frac{x}{\sqrt{3}}$, SO 3) $A = \pi\left(\frac{x}{\sqrt{3}}\right)^2 \implies \boxed{A = \frac{\pi x^2}{3}}$ (DOMAIN $= (0,\infty)$)

OR



1) $A = \pi r^2$

2) $y^2 + \left(\frac{x}{2}\right)^2 = r^2$ BY THE PYTHAGOREAN TH., AND $y = \frac{r}{2}$

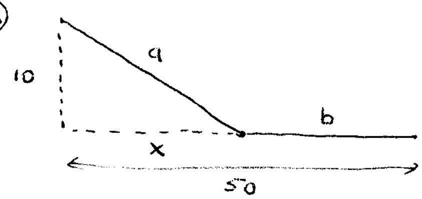
SINCE y IS THE SIDE OPPOSITE THE 30° ANGLE IN A 30°-60°-90° Δ;

SO $\left(\frac{r}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = r^2 \implies \frac{r^2}{4} + \frac{x^2}{4} = r^2 \implies r^2 + x^2 = 4r^2$

$\implies x^2 = 3r^2 \implies r^2 = \frac{x^2}{3}$.

THEN 3) $A = \pi r^2$ GIVES $\boxed{A = \frac{\pi x^2}{3}}$ (DOMAIN $= (0,\infty)$)

44a



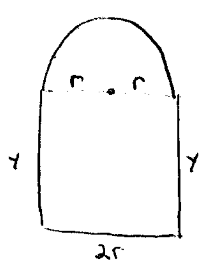
1) $C = 8000a + 2000b$ (SINCE COST $= \frac{\text{COST}}{\text{DISTANCE}} \cdot$ DISTANCE)

2) $a^2 = x^2 + 10^2$ SO $a = \sqrt{x^2 + 100}$

$x + b = 50$ SO $b = 50 - x$

3) $\boxed{C = 8000\sqrt{x^2+100} + 2000(50-x)}$ (DOMAIN $= [0, 50]$)

47a



1) $A = 2ry + \frac{1}{2}\pi r^2$

2) $P = 2r + 2y + \frac{1}{2}(2\pi r) = 2r + 2y + \pi r = 2y + (2+\pi)r = 32$,

SO $2y = 32 - (2+\pi)r$

3) $A = r(32 - (2+\pi)r) + \frac{1}{2}\pi r^2 = 32r - (2+\pi)r^2 + \frac{1}{2}\pi r^2$

$= 32r - \left[(2+\pi) - \frac{1}{2}\pi\right]r^2$, SO

$\boxed{A = 32r - \left(2 + \frac{\pi}{2}\right)r^2}$ (DOMAIN $= \left(0, \frac{32}{2+\pi}\right)$ SINCE $r > 0$ AND $y > 0$

4.5 – 10 $h = 512t - 16t^2$

a) IT REACHES MAX. HEIGHT WHEN $t = -\frac{b}{2a} = -\frac{512}{2(-16)} = 16$ SEC, SO

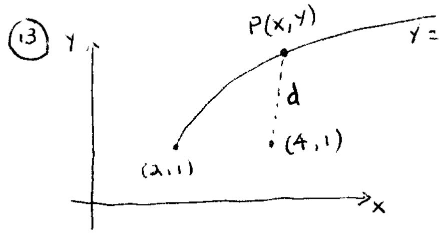
$h(16) = 512(16) - 16(16)^2 = 256(32-16) = \boxed{4096 \text{ FT}}$ IS THE MAX. HEIGHT

b) IT HITS THE GROUND WHEN $h = 0$, SO

$512t - 16t^2 = 0 \implies 16t(32 - t) = 0 \implies \cancel{t=0}$ OR $\boxed{t = 32 \text{ SEC}}$

MATH 12 SEC. 4.5 – SOLUTIONS

(13)



1) MINIMIZE $d = \sqrt{(x-4)^2 + (y-1)^2}$ OR, EQUIVALENTLY,

$$d^2 = (x-4)^2 + (y-1)^2$$

2) $y = \sqrt{x-2} + 1$, SO

3) $d^2 = (x-4)^2 + (\sqrt{x-2} + \cancel{1} - \cancel{1})^2$

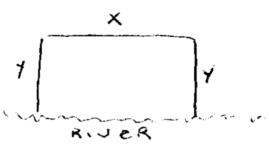
$= x^2 - 8x + 16 + x - 2 = x^2 - 7x + 14$

4) d^2 HAS A MIN. WHEN $x = -\frac{b}{2a} = -\frac{-7}{2(1)} = \frac{7}{2}$ AND $y = \sqrt{\frac{7}{2} - 2} + 1 = \sqrt{\frac{3}{2}} + 1 = \frac{\sqrt{6}}{2} + 1$,

SO $\boxed{\left(\frac{7}{2}, \frac{\sqrt{6}}{2} + 1\right)}$ IS THE CLOSEST POINT.

WHEN $x = \frac{7}{2}$, $d^2 = x^2 - 7x + 14 = \frac{49}{4} - \frac{49}{2} + 14 = -\frac{49}{4} + \frac{56}{4} = \frac{7}{4}$ SO $d = \sqrt{\frac{7}{4}} = \boxed{\frac{\sqrt{7}}{2}}$

(17)



1) $A = xy$ (MAXIMIZE)

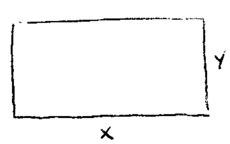
2) $x + 2y = 500$, SO $x = 500 - 2y$

3) $A = (500 - 2y)y = 500y - 2y^2$

4) A HAS A MAX. WHEN $y = -\frac{b}{2a} = -\frac{500}{2(-2)} = \boxed{125 \text{ FT}}$

AND $x = 500 - 2(125) = \boxed{250 \text{ FT}}$

(35)



1) MAXIMIZE $A = xy$

2) $12(2x) + 8(2y) = 4800$, SO $24x + 16y = 4800$,

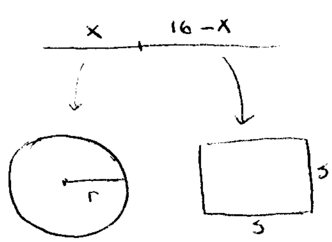
$3x + 2y = 600$, $2y = 600 - 3x$, $y = 300 - \frac{3}{2}x$

3) $A = x(300 - \frac{3}{2}x) = 300x - \frac{3}{2}x^2$

4) A HAS A MAX. WHEN $x = -\frac{b}{2a} = -\frac{300}{2(-\frac{3}{2})} = \boxed{100 \text{ YD}}$

AND $y = 300 - \frac{3}{2}(100) = \boxed{150 \text{ YD}}$

(45)



1) $A = \pi r^2 + s^2$

2) $2\pi r = x$, SO $r = \frac{x}{2\pi}$

$4s = 16 - x$, SO $s = \frac{16 - x}{4} = 4 - \frac{x}{4}$

3) $A = \pi\left(\frac{x}{2\pi}\right)^2 + \left(4 - \frac{x}{4}\right)^2 = \pi\left(\frac{x^2}{4\pi^{\cancel{2}}}\right) + \left(16 - 2x + \frac{x^2}{16}\right)$

SO $A = \frac{x^2}{4\pi} + 16 - 2x + \frac{x^2}{16} = \left(\frac{4x^2}{16\pi} + \frac{\pi x^2}{16\pi}\right) - 2x + 16$

OR a) $\boxed{A = \left(\frac{4 + \pi}{16\pi}\right)x^2 - 2x + 16}$

b) 4) A HAS A MIN. WHEN $x = -\frac{b}{2a} = -\frac{-2}{2\left(\frac{4+\pi}{16\pi}\right)} = \boxed{\frac{16\pi}{4+\pi} \text{ IN}}$