MATH 12 SOLUTIONS – SEC. 8.1 AND 9.1

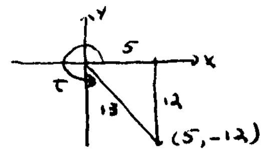
8.1 – (26) $\cos t = \frac{5}{13}$, $\frac{3\pi}{2} < t < 2\pi$

$\sin^2 t = 1 - \cos^2 t = 1 - \frac{25}{169} = \frac{144}{169}$, so $\boxed{\sin t = -\frac{12}{13}}$ (since $\sin t < 0$ in Quad. IV)

Then $\boxed{\cot t} = \frac{\cos t}{\sin t} = \frac{5/13}{-12/13} = \boxed{-\frac{5}{12}}$

OR $\cos t = \frac{5}{13} = \frac{x}{r}$, so let $\underline{x = 5}$ and $\underline{r = 13}$.

Then $x^2 + y^2 = r^2 \Rightarrow 25 + y^2 = 169 \Rightarrow y^2 = 144 \Rightarrow y = -12$



$\boxed{\sin t} = \frac{y}{r} = \boxed{-\frac{12}{13}}$

$\boxed{\cot t} = \frac{\cos t}{\sin t} = \frac{5/13}{-12/13} = \boxed{-\frac{5}{12}}$ (or use $\cot t = \frac{x}{y}$)

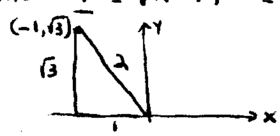
(30) $\cot\theta = -\frac{1}{\sqrt{3}}$, $\cos\theta < 0$ (so θ is in Quad. II)

$\csc^2\theta = 1 + \cot^2\theta = 1 + \frac{1}{3} = \frac{4}{3}$, so $\boxed{\csc\theta = \frac{2}{\sqrt{3}}}$ (since $\csc\theta > 0$ in Quad. II)

and $\boxed{\sin\theta = \frac{\sqrt{3}}{2}}$ since $\sin\theta = \frac{1}{\csc\theta}$

OR $\cot\theta = \frac{x}{y} = -\frac{1}{\sqrt{3}}$, so let $\underline{x = -1}$ and $\underline{y = \sqrt{3}}$ (since θ is in Quad. II)

Then $r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = \sqrt{4} = \underline{2}$, so $\boxed{\sin\theta} = \frac{y}{r} = \boxed{\frac{\sqrt{3}}{2}}$



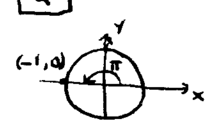
and $\boxed{\csc\theta = \frac{2}{\sqrt{3}}}$ since $\csc\theta = \frac{1}{\sin\theta}$

(34) $\frac{1}{(x^2+5)^2} = \frac{1}{((\sqrt{5}\tan\theta)^2 + 5)^2} = \frac{1}{(5\tan^2\theta + 5)^2} = \frac{1}{(5(\underline{\tan^2\theta + 1}))^2} = \frac{1}{(5\underline{\sec^2\theta})^2}$

$= \frac{1}{5^2\sec^4\theta} = \boxed{\frac{1}{25}\cos^4\theta}$ $\left(\text{since } \frac{1}{\sec\theta} = \cos\theta\right)$

(42) a) $\sin\frac{17\pi}{4} = \sin\left(2(2\pi) + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \boxed{\frac{1}{\sqrt{2}}} = \boxed{\frac{\sqrt{2}}{2}}$ since $\frac{17\pi}{4} \div 2\pi = \frac{17\pi}{4}\cdot\frac{1}{2\pi} = \frac{17}{8} = 2 + \frac{1}{8}$

c) $\cos 11\pi = \cos(5(2\pi) + \pi) = \cos\pi = \boxed{-1}$

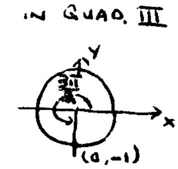


since $\frac{11\pi}{2\pi} = \frac{11}{2} = 5 + \frac{1}{2}$

d) $\cos\frac{53\pi}{4} = \cos\left(6(2\pi) + \frac{5}{8}(2\pi)\right) = \cos\frac{5\pi}{4} = -\cos\frac{\pi}{4} = \boxed{-\frac{1}{\sqrt{2}}} = \boxed{-\frac{\sqrt{2}}{2}}$

since $\frac{53\pi}{4} \div 2\pi = \frac{53\pi}{4}\cdot\frac{1}{2\pi} = \frac{53}{8} = 6 + \frac{5}{8}$, $\theta^* = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$, and $\cos\theta < 0$ in Quad. III

9.1 – (12) $\cos\left(\frac{3\pi}{2} + \theta\right) = \cos\frac{3\pi}{2}\cos\theta - \sin\frac{3\pi}{2}\sin\theta = 0\cdot\cos\theta - (-1)\sin\theta = \boxed{\sin\theta}$



(18) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

MATH 12 SOLUTIONS – SEC. 9.1 AND 9.2 AND YELLOW SHEET

9.1 – 20 a) $\sin 105° = \sin(60° + 45°) = \sin 60° \cos 45° + \cos 60° \sin 45°$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

25 $\sin\alpha = \frac{12}{13}$, $\frac{\pi}{2} < \alpha < \pi$ AND $\cos\beta = -\frac{3}{5}$, $\pi < \beta < \frac{3\pi}{2}$

$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{144}{169} = \frac{25}{169}$, so $\cos\alpha = -\frac{5}{13}$ (since $\cos\theta < 0$ in Q. II)

$\sin^2\beta = 1 - \cos^2\beta = 1 - \frac{9}{25} = \frac{16}{25}$, so $\sin\beta = -\frac{4}{5}$ (since $\sin\theta < 0$ in Q. III)

a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \left(\frac{12}{13}\right)\left(-\frac{3}{5}\right) + \left(-\frac{5}{13}\right)\left(-\frac{4}{5}\right) = -\frac{36}{65} + \frac{20}{65} = \boxed{-\frac{16}{65}}$

b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(-\frac{5}{13}\right)\left(-\frac{3}{5}\right) - \left(\frac{12}{13}\right)\left(-\frac{4}{5}\right) = \frac{15}{65} + \frac{48}{65} = \boxed{\frac{63}{65}}$

9.2 – 4 $\cot s = 2$, $\pi < s < \frac{3\pi}{2}$

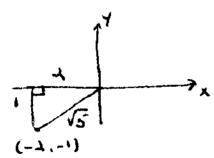
$\csc^2 s = \cot^2 s + 1 = 4 + 1 = 5$, so $\csc s = -\sqrt{5}$ (since $\csc\theta < 0$ in Q. III), THEN

$\sin s = \frac{1}{\csc s} = -\frac{1}{\sqrt{5}}$, AND $\cos s = \left(\frac{\cos s}{\sin s}\right) \cdot \sin s = (\cot s)(\sin s) = 2\left(-\frac{1}{\sqrt{5}}\right) = -\frac{2}{\sqrt{5}}$

a) $\sin 2s = 2\sin s\cos s = 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \boxed{\frac{4}{5}}$

b) $\cos 2s = \cos^2 s - \sin^2 s = \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$

OR use $\cot s = \frac{x}{y} = 2$, so LET $x = -2$ AND $y = -1$ (since s is in Q. III),
THEN $x^2 + y^2 = r^2$ gives $r^2 = 4 + 1 = 5$, so $r = \sqrt{5}$ (since $r > 0$).
THEN $\cos s = \frac{x}{r} = -\frac{2}{\sqrt{5}}$ AND $\sin s = \frac{y}{r} = -\frac{1}{\sqrt{5}}$



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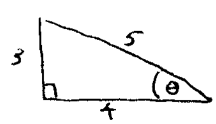

a) $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{\frac{24}{25}}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

Y.S. – 3 $\frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5}{3}x^{-1/3}(x^1 - 2) = \boxed{\frac{5}{3}\left(\frac{x-2}{x^{1/3}}\right)} = \boxed{\frac{5(x-2)}{3x^{1/3}}}$

OR $\frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5}{3}\left(x^{2/3} - \frac{2}{x^{1/3}}\right) = \frac{5}{3}\left(x^{2/3} \cdot \frac{x^{1/3}}{x^{1/3}} - \frac{2}{x^{1/3}}\right) = \boxed{\frac{5}{3}\left(\frac{x-2}{x^{1/3}}\right)} = \boxed{\frac{5(x-2)}{3x^{1/3}}}$

4 $x \cdot \frac{1}{\cancel{2}}(2x+3)^{-1/2} \cdot \cancel{2} + \sqrt{2x+3} = (2x+3)^{-1/2}\left[x + (2x+3)^1\right] = \frac{3x+3}{(2x+3)^{1/2}} = \boxed{\frac{3(x+1)}{(2x+3)^{1/2}}}$

OR $x \cdot \frac{1}{\cancel{2}}(2x+3)^{-1/2} \cdot \cancel{2} + \sqrt{2x+3} = \frac{x}{(2x+3)^{1/2}} + (2x+3)^{1/2} \cdot \frac{(2x+3)^{1/2}}{(2x+3)^{1/2}}$

$= \frac{x}{(2x+3)^{1/2}} + \frac{2x+3}{(2x+3)^{1/2}} = \frac{3x+3}{(2x+3)^{1/2}} = \boxed{\frac{3(x+1)}{(2x+3)^{1/2}}} = \boxed{\frac{3(x+1)}{\sqrt{2x+3}}}$